A RATIONAL-CONSTRUCTIVIST ACCOUNT OF EARLY LEARNING ABOUT NUMBERS AND OBJECTS

Rochel Gelman

I. Introduction

This article features my rational-constructivist account of cognitive development. The rationalist side of the theory captures the assumption that our young bring a skeletal outline of domain-specific knowledge to their task of learning the initial concepts they will share with others. The constructivist side of the theory captures the assumption that, from the start, our young actively join in their own cognitive development. Even as beginning learners, skeletal principles motivate them to seek out and assimilate inputs that nurture the development of these structures. To develop these assumptions I consider work on two topics: (1) conceptions of objects during infancy and (2) numerical concepts in infants and beginning language users. Special attention is given to the need to consider whether differences in performance levels across tasks are due to limits on the conceptual competence under investigation or to limits on the procedural and interpretative competences needed for successful performance.

There is no a priori reason to assume that the rational and the constructivist positions are inconsistent or contradictory. I join Marler in his challenge of those who still "think of learning and instinct as being virtually antithetical . . . [that] behavior is one or the other, but not both" (p. 37, 1991). In the history of science, key terms shifted their meaning when

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understanding of the phenomena to which they refer changed. For example, developments in physics, mathematics, and biology led to changes in the meaning of *movement, zero,* and *alive* (see Carey, 1985; Kitcher, 1982; Kuhn, 1970; Mayer, 1982; and Wiser, 1987). Likewise, recent advances in neuroscience, animal learning, and ethology are producing shifts in the meaning of phrases or terms like *biological underpinnings* and *innate*. The more we understand about the acquisition of complex actions, the more we appreciate that they depend on organisms' opportunities to interact with and assimilate relevant environments. To say that genetic history contributes to the development of some classes of behavior is not to say that these will appear full-blown at a given point in time. Without opportunities to engage with and learn about the kinds of environments that nurture the potential given by the genetic history, development either will be abnormal or will fail to occur. Normal development is intricately tied to opportunities to interact with and process relevant environments. The story of how the young male white-crowned sparrow comes to learn his species-specific adult song provides an elegant example of these points.

The male white-crowned sparrow is born with a template specifying basic features of his adult song. However, he must hear examples of the correct conspecific song during a critical period early in development. If he is reared in an environment that does not include examples of his adult dialect, he is able to learn a nonpreferred song. Therefore, the learning process that supports acquisition of the conspecific song can yield unexpected or inappropriate outcomes, given sufficient atypical experience. Nevertheless, "errors" seldom occur because learning typically takes place in a supporting ecology.

Similarly, the opportunity to interact with the environment supports another key step in song development. The initial song, which is first produced well after the above critical period, is far from the adult song. Production of the adult crystallized song is preceded by a lengthy trial-and-error period. Although the bird does not have to hear any further inputs from other birds during this period, it appears that he does have to hear himself produce what are called *subsongs* and *plastic songs*. It seems as if the remembered song provides birds with a standard against which to compare their output, much as memories of recordings or performances can aid music students as they practice.

Such examples help to illustrate how the meaning of terms and phrases like *learn, innate,* and *biological contributions* are changing. Indeed, Marler (1991) now writes of "the instinct to learn" and refuses to pit terms like *practice, trial-and-error, variability,* and *learn* against ones like *constraint, innate, biological, genetic,* and so on. Parallel shifts in meaning can be found in writings about animal learning (Gallistel, 1990; Rozin &

Schull, 1988) and cognitive development (e.g., Carey & Gelman, 1991; Karmiloff-Smith, 1992; Keil, 1981). These developments serve as the backdrop for my rational–constructivist account of knowledge acquisition. I have been especially concerned with the specification of the nature of relevant inputs and the laws of learning that apply for such an account.

II. Different Accounts of Initial Concepts

A. Association Theories of Learning

There have been important developments in associative accounts of learning, especially regarding the need for the conditioned stimulus (CS) to predict the unconditioned stimulus (UCS) (Rescorla & Wagner, 1972). Still, the empiricists' assumptions about the acquisition of knowledge remain as core assumptions in modern associationist accounts of concept development and learning. These assumptions are that all knowledge can be traced to our ability to process sensory inputs and to form associations between these sensations (S-S connections) and/or to our responses to these sensations (S-R connections). In the case of the infant, what is given is the ability to receive punctate sensations of light, or sound, or pressure, and so forth, and to form associations between these according to the laws of association (frequency and proximity). Sensations and responses that occur close together (in time or space) and repeatedly are more likely to be associated than those that are infrequent and far apart. As associations between sensations and responses are impressed on the infant's blank mental slate, these too become associated with incoming data or each other and lay the groundwork for knowledge of the world at a sensory and motor level. These further associations in turn support knowledge acquisition at the perceptual level. Experiences at the perceptual level provide the opportunity for cross-modal associative learning and thus for the eventual induction of abstract concepts that cut across concepts about particular perceptual information.

B. Developmental Theories of Learning

Developmental textbooks often pit learning theoretic (read as "associationist") accounts against developmental ones. In this context, the idea is that development involves more than "mere" learning. Cognitive development proceeds through stages, and the way learners interpret inputs of a given kind is influenced by the stage they have achieved. For example, during the first two years of life, Piaget's sensorimotor infants can build schemes relating actions to what they see, hear, touch, and so on; however, they will not be able to represent a set of objects in terms of class inclusion until they reach concrete operations at about 6 to 9 years of age.

Paired with the assumption of stages is the related assumption that learners actively interpret inputs with reference to their available knowledge and mental structures. In the Piagetian framework, the construction of the "correct" interpretation of the transformations performed on quantities must await the child's advance to concrete operational thought. The younger child's belief that the amount of water in a glass changes as it is poured into another, different-shaped glass, reflects reliance on perceptual information (Piaget, 1952).

As we shall see, there are important differing foundational assumptions of the associationist and developmental accounts of infant-knowledge acquisition. Still, the two classes of characterizations of an infant's initial world are more similar than not. For example, Piaget limits an infant's initial knowledge to a level that is controlled reflexively. The active practice of these reflexes leads to their adaptation into sensorimotor schemes. The active use of the consequent scheme leads to the development of intercoordinated schemes of action. The more such intercoordinations, the more likely that the infant builds a world of three-dimensional objects in a three-dimensional space.

In the associationist account, infants gradually build up a notion of an individual object by associating the primitive sensations generated by different objects. Somehow, by forming associative clusters for many different objects, young learners eventually produce the concept of an object as something that exclusively occupies a volume of space at a particular time and that has properties such as color, shape, weight, and so on.

To be sure, associations are not Piaget's fundamental building blocks of cognition; sensorimotor schemes are. Nevertheless, his infant must have repeated interactions with objects in order to achieve more coordinated memories among those sensorimotor schemes that are used with a given object. These action-based representations help move the infant from a state of out-of-sight, out-of-mind to states that lead to a concept of an object. Only then does the infant finally know that an object persists over time and in space, whether or not it is covered and/or moved through invisible displacements.

Some of Piaget's foundational assumptions about the nature of the data required to drive development apply to the developmental theories advanced by Bruner, Vygotsky, and Werner. In each case, learners are assumed to approach objects on the basis of simple motoric, sensory, or perceptual features. Given suitable opportunities to develop further knowledge of these features, the child is able to induce more abstract concepts. Initial "concepts," whose core consists of sensorimotor associations or perceptual rules of organization, are variously labeled as graphic collections, preconcepts, complexes, pseudo-concepts, and chain concepts.

In sum, whether the account of the origins of knowledge is rooted in an associationist view or one of the classical theories of cognitive development, the assumption is that first-order sense data, for example, sensations of colored light, sound, pressure, and so forth, serve as the foundation upon which knowledge is developed. Principles that organize the buildup of representations of concepts, as a function of experience and the opportunity to form associative networks or sensorimotor schemes, are induced.

C. More on the Rational-Constructivist Account

A key assumption that I make is that cognitive development is channeled by innate, domain-specific principles—even in infants. Another is that infants build models of the world in accord with these first principles. What follows expands on these notions. As we shall see, this view has implications regarding which data are relevant for concept acquisition. In order to show why this is so, it helps to have a way to identify a domain.

1. *Defining a Domain of Knowledge*

I define a domain of knowledge in much the same way that formalists do, by appeal to the notion of a set of interrelated principles. A given set of principles, the rules of their application, and the entities to which they apply together constitute a domain. Because different structures are defined by different sets of principles, we can say that a body of knowledge constitutes a domain of knowledge to the extent that we can show that a set of interrelated principles organize the entities and related knowledge as well as the rules of operation on these. Note that there is nothing in this definition that requires that domain-specific knowledge be built on an innate foundation. Whether a domain is acquired or not is an orthogonal issue.

Counting is a part of a number-specific domain, because the representatives of numerosity (what I call "numerons") generated by counting are operated on by mechanisms informed by, or obedient to, arithmetic principles. For counting to provide the input for arithmetic reasoning, the principles governing counting must complement the principles governing arithmetic reasoning. For example, the counting principles must be such that sets assigned the same numeron are in fact numerically equal and the set assigned a greater numeron is more numerous than a set assigned a lesser numeron. Similarly, the analysis of the causation of movements is based on a domain-specific set of principles. This is the case because there are principles that govern reasoning about the causes of motion and perceptual mechanisms that are informed by complementary principles, those that recognize different categories of causation for the movements of biological objects as a whole as opposed to the movements of inanimate objects as a whole (Gelman, 1990).

In contrast, general processes, like discrimination, or general-purpose processing mechanisms, like short-term memory, do not constitute domains any more than the process of applying rewrite rules, which is common to all formal systems, constitutes a domain of mathematics. Nor does a script structure constitute a domain. Scripts are analogous to the heuristic prescriptions for solving problems in mathematics, which should not be confused with the mathematical domains themselves (algebra, calculus, theory of functions, etc.).

In sum, when we suspect that something is a conceptual domain, we can test whether this is so by seeing whether it is possible to characterize it in terms of a coherent set of operating principles and their related entities. I must emphasize an important aspect about the definition of a knowledge domain that I favor. This is that it is neutral about whether it starts as an innate skeletal set of principles or whether it is learned from scratch. For a related reason there is no necessary link between the idea that knowledge is domain specific and when the knowledge within the domain is acquired. Given my definition of a domain, expert chess players possess domain-specific knowledge of chess. Yet, surely they were not endowed with innate knowledge of the structure that organizes the domain, the entities, and the permissible moves. Similarly, my attribution of some domain specific knowledge to infants does not block me from using domain-general learning principles, that is, ones that apply across domains as part of my account of cognitive development.

2. *On Granting Innate Knowledge of a Domain*

What does it mean to say that some domains of knowledge are innately given? When I postulate that early cognitive development is directed by domain-specific principles, I find it helpful to use the metaphor of a skeleton. Were there no skeletons to dictate the shape and contents of the bodies of pertinent knowledge, then the acquired representations would not cohere. Just as different skeletons are assembled according to different principles, so too are different coherent bodies of knowledge. Skeletons need not be evident on the surface of a body. Similarly the underlying axiom-like principles that enable the acquisition of coherent knowledge need never be accessible.

The aptness of the skeleton metaphor is less than perfect. It carries the implication that all principles are in place before their respective bodies of knowledge are acquired. This is unlikely. In fact, it is possible that only some subset of principles of a domain serve as part of the initial skeleton. Further, initial principles might even be replaced or expanded over the course of learning, especially if the learner acquires new or extremely enriched theories (Carey, 1985), or has the capacity for mapping language onto principles that are not at first statable or even symbolically represented (Karmiloff-Smith, 1986).

3. *Adding the Constructivist Side of the Theory*

We assume that all learners actively apply their available structures, no matter how young they are. If we allow that infants use skeletal structures, we account for their tendencies to attend selectively and to respond in structurally coherent ways to their environment. The knowledge that infants acquire when they actively apply their nascent skeletal structures to the world feeds the development of these structures. This occurs because the principles function both to define the constraints on the class of relevant inputs and to store, in a coherent fashion, experiences with these inputs. As relevant inputs are assimilated to these principles, they feed the coherent development of knowledge within their respective domains.

The principles that define a domain of conceptual development need not be represented within the system in a symbolic or linguistic form. Rather they can be, and most likely are, first represented within the structure of the information-processing mechanisms that assimilate experience and direct action (cf. Karmiloff-Smith, 1992). Marr (1982) presents many cases where the algorithms by which the visual system processes visual input implicitly incorporate various principles about the structure of the world. Gallistel, in 1990 (see also Cheng & Gallistel, 1984), argued that the principles of Euclidean geometry are implicit in the mechanisms by which the rat constructs and uses a map of its environment. Knudsen's (1983) work on the development of the tectal circuitry for representing the angular positions of distal stimuli apprehended by different sensory modalities in the barn owl provides a clear example of how a principle can be implicit in a developmental mechanism. Implicit in the mechanism that controls the development of tectal circuitry of the owl is the principle that the spatial matrix for experience is unitary and transcends sensory modality. An object cannot have one location in the space apprehended through the visual modality and a different location in the space apprehended through the auditory modality. Thus, when the mapping of visual locations is experimentally put out of register with the mapping of auditory locations, the maturing circuitry reorganizes so as to bring the mappings back into register.

Because skeletal principles give the young constructivist mind a way to attend to and selectively process data, they similarly contribute to the nurturing and development of the concepts that limn the domain in question. No matter how skeletal these first principles may be, they still can organize the search for, and assimilation of inputs that can feed the development of the concepts of the domain. Actively assimilated inputs help flesh out the skeletal structure. The more structured knowledge there is, the more it is possible for the learner to find domain-relevant inputs to attend to and actively assimilate to the existing structure. The positive feedback set up underlies the continual buildup of the knowledge structure within the domain. For a related discussion of structure mapping in vision see Bedford (this volume).

In sum, first principles help focus attention on inputs that are relevant for the acquisition of concepts in their domain. It matters not that these first principles are implicit, preverbal and sketchy in form. What matters is that they are structured. Their active application leads infants and young children to find and store experience in domain-relevant ways.

The above has implications for what counts as primary data within a rational-constructivist account of concept acquisition. Since the relevant primary data are defined in terms of abstract principles, they must be relational. Simple sensations no longer are the basic data for concept acquisition. If infants come to the world with both skeletal knowledge and a tendency to apply it, we should expect them to attend selectively to domain-relevant inputs that are appropriately structured. If, for example, infants are endowed with principles that focus their attention on things in the world, they might attend to a three-dimensional thing before attending to its color. Similarly, infants might take a set of things as an opportunity to apply implicit nonverbal counting principles before they attend to whether the things are the same color or shape. These possibilities, of course, reverse the assumptions that associationist theories make about what is initially salient and pertinent to both perceptual and cognitive development. Such "possibilities" are in fact more than possible outcomes. They are well-established results. The pertinent studies took advantage of infants' tendencies to explore their environments with whatever responses they have in their limited repertoire of behaviors. Infants look, listen, and even suck more when presented with a stimulus of interest. The rates or strength of these responses decline as the novel becomes familiar.

Infants resume responding when they detect a change in the input. Thus, the tendency to dishabituate can index an infant's ability to differentiate between different input conditions. This all supports the assumption of a constructivist infant mind—one that is data seeking and data integrating. The fact that these tendencies often are applied to structurally complex data motivates a theory that pairs constructivism with rationalism to generate an infant mind that assimilates data in accord with abstract principles.

Piaget and others using his procedures have repeatedly found that 4- to 8-month olds will reach for and even grasp a toy only as long as they can see it. They stop reaching or looking as soon as the toy disappears behind a barrier. Their failure to continue to be interested in objects once they are covered led Piaget to conclude that young infants lack a representation of an object. They behave as if governed by an "out of sight, out of mind" epistemology. The long-standing assumption that young infants do not represent objects has been challenged by two series of studies, one from Baillargeon's lab and one from Spelke's. These investigators offer evidence that infants come to the world prepared to behave as if there are three-dimensional objects that occupy space and do not move through each other. The contrast between Piaget's conclusions, on the one hand, and Baillargeon's and Spelke's, on the other hand, provides an especially compelling case example of how much displays of competence can vary across tasks.

In Baillargeon, Spelke, and Wasserman (1985), 6- to 8-month-old infants were first shown a screen as it rotated toward and away from them, through an 180° arc. When their interest in the moving screen declined, that is, when the infants habituated, they were shown a new display. This consisted of an object that was held stationary in an upright position to the left side of the screen. Once an infant looked at the object, the experimenter moved it behind the screen. This set the stage for the post-habituation phase of the experiment. Once again the screen rotated toward and away from the infant. Given the physics of the situation, the screen should have stopped at about the 120° position of its rotation. When it continued through a 180° arc (thanks to the use of trick mirrors or trapdoors), it contributed to the adult perception of an impossible event. It looked as if the (unseen) block behind the screen were repeatedly crushed and uncrushed as the screen rotated forward and backward through a 180° arc. The event is impossible for adults because for them—save in the world of spirits and ghosts—one solid object cannot pass through another one. If, as Piaget suggests, infants this young believe that objects disappear once they are no longer in view, then the 180° rotation event should not bother them. Indeed, they should continue to be bored by the screen that passes through a full 180° arc. Instead, they should treat as more novel the event in which the screen rotates through only a 110° arc. However, they did not. The infants in the experiment in fact looked more at the impossible event. Therefore, they must have reinterpreted the 180° arc event. Otherwise they could not have "seen" a different event, one in which a screen seemed to go through (or crush) the (hidden) object.

Baillargeon (1986) also has shown that 6- to 8-month-old infants are surprised when a train that moves behind a barrier follows a trajectory that passes right through a rigid object. Unseen events serve to achieve what looks like an impossible outcome. Spelke (1991) provides similar evidence with still younger infants. She first ran 3- to 4-month-old infants in a three-step habituation event. To start each habituation trial, a ball was held up in the air over a screen. It was then dropped behind the screen. Finally, the screen was removed to reveal the ball resting on the surface of a table. This event sequence was repeated until infants habituated, that is, until they looked less than half as long during a trial as they once did. During the post-habituation phase, the event sequence was the same for its first two steps; in the final, third step, infants viewed the object resting either on top of or under a novel shelf that had been placed surreptitiously into the display. For the latter case, the ball ended up in a familiar position, on top of the table. However, to get there, it would have had to pass through the shelf that sat between the point and the table. Therefore, although both of the post-habituation events were novel, only the latter (on-the-table) one was impossible. Once again, infants looked longer at the impossible events. In a control set of conditions, infants saw the exact same final displays in the post-habituation phase, but the balls were not dropped. Therefore, neither outcome was impossible. Now infants preferred to look at the ball that was resting in a novel place, that is, on the shelf, as opposed to on the floor (and under the shelf). This pattern of results suggests that even some 3- to 4-month-olds care about impossible events, a possibility that gains credence given that Baillargeon (1987) finds that a reliable number of a sample of 3½- to 4½-month-olds respond to the rotating screen conditions as did the older infants in Baillargeon et al. (1985).

Spelke has appealed to findings like the above to conclude that, from the start, infants behave as if the world has three-dimensional things "out there," things that occupy space and maintain both their coherence and boundaries as they move. Principles of object perception can support this outcome. Additional conceptual principles are needed to account for infants' reactions to the presentation of impossible physical events. The principles of perception that can lead infants to find three-dimensional things are as follows: (1) Two surfaces will be perceived as part of the same object if they touch each other; and (2) two surfaces that move together at the same time and speed along parallel paths in three-dimensional space—even if their connection is concealed—will be perceived as the surface of the same object. The ability to react correctly (with surprise or renewed attention) to the seeming ability of objects to pass through each other suggests that infants also have implicit conceptual principles about objects, including that they are solid.

The foregoing perceptual principles are neutral as to whether infants can or cannot perceive color, shape, and size. They are neutral because the principles do not need to use such information to support the ability to find three-dimensional things. Therefore, these primary sense data are not foundational in the Spelke account of how infants perceive three-dimensional objects. It is not that Spelke denies infants the ability to use sensations like these. Rather, her view is that sensory abilities work in the service of the perceptual principles. Once objects are located, infants then learn about their attributes, what goes with what, and so on. Infants could even count these things, whether or not they have learned about their characteristic attributes, because counting principles are indifferent to the attributes of objects.

My suggestion that infants, or anyone, might count objects even if they know nothing or very little about these things is inconsistent with the associationist, stage theoretic and logicist account of number. In all of the latter accounts, number concepts are treated as higher and later-learned abstractions. It is therefore not surprising that those who favor these theories of number concepts are the same individuals who offer non-numerical accounts of infants' ability to respond to the numerical information in displays.

There are many demonstrations that infants attend to the numerical information in displays (Cooper, 1984; Sophian & Adams, 1987; Starkey, 1992; Starkey Spelke, & Gelman, 1983, 1990; Strauss, 1984; Wynn, 1992a). For example, Starkey et al. (1990) show that infants prefer to look at a slide that contains the same number of heterogeneous household items (two or three) that they hear in a sequence of drum beats (two or three). Infants of 12 months or younger also keep track of the effects of surreptitious additions and subtractions (Starkey, 1992; Wynn, 1992a), which means that they can order the set sizes they encounter (see also, Baillargeon, Miller, & Constantino, 1992; Sophian & Adams, 1987; Strauss & Curtis, 1984).

Additionally, infants respond to numerosity when the items in a set each move on separate paths during a trial (van Loosbroek & Smitsman, 1990), that is, when any pattern is obscured. They also match the number of heterogeneous items they see with the number of drum beats they hear, whether or not time is constant or varied across the drum-beat events (Starkey, Spelke, & Gelman, 1990). Whatever the account of these findings, it must deal with infants' ability to respond amodally to numerical information. The next section focuses on possible explanations, including my rational–constructivist explanation. Like Spelke and Baillargeon, I do *not* claim full-blown knowledge, either of numbers or of objects and their properties. All of us are concerned with what is learned, as well as with explanations of the difference between our findings and those of others, especially Piaget's. We are particularly interested in developing a learning theory that explains systematic across-task and within-tack variability in performances that depend on the same conceptual competence (Gelman, 1991; Gelman, Massey, & McManus, 1991).

III. On Variability

Some authors argue that findings like those reviewed above are obtained under too limited a set of conditions and therefore do not justify the attribution of principled knowledge about objects and numerosity. For example, Fischer and Biddle (1991) take the fact that 4- to 8-month-old infants fail even the simple Piagetian covering tasks as compelling reason to reject Baillargeon's and Spelke's attributions of conceptual competence for objects to such very young infants. To be sure, systematic within- and across-condition variability in the extent to which performance conforms to abstract principles is consistent with traditional learning and developmental theories. For both classes of theories, unprincipled "habits" are acquired prior to the induction of principles. However, contrary to widespread assumption, rational accounts of cognitive development do not predict errorless performance from the start. It therefore behooves us to consider more carefully the way systematic cross-task variability is treated in different accounts of concept development.

Gelman and Greeno (1989) point out that there are a number of systematic sources of variability that can mask conceptual competence, including limited procedural and interpretative competence. Because Gallistel and Gelman's (1992) competence model of preverbal counting makes use of mechanisms whose outputs are inherently variable, it is also necessary to find ways to relate details of variability at this level to choices of models.

In their 1989 paper Gelman and Greeno expand on their initial proposal (Greeno, Riley & Gelman, 1984) that competent plans of action require the successful integration of *conceptual, procedural* (planning), and *interpretative* (utilization) competence. A competent plan of action must honor the constraints of conceptual competence. For example, for a plan of counting to be competent, it must incorporate the constraints of the one-for-one counting principle. The plan must not embrace component acts of doubletagging, item-skipping, or tag repeating. Additionally, the plan has to be responsive to constraints on the interpretations of the task setting, instructions, domain-related terms, conversational rules, and so forth. The limited development or misapplication of setting relevant conversational rules can lead to faulty plans of action in a given setting and therefore to variability in success levels across studies or tasks. This possibility is illustrated in Gelman, Meck and Merkin's (1986) use of the doesn't-matter counting task that asked children to count a row of items in a novel way.

The doesn't-matter task begins when the experimenter points to an object that is not at an end of a row of items and asks the child to make that object "the one" and to count all of the objects. To accomplish this, a child has to skip back and forth over the items while counting, switch the designated item with one that is at an end, or count as if the row of items were in a circle. Interestingly, very young children who were given a chance to count a row of items before they started the doesn't-matter task did more poorly than children who had no pretest counting experience. Inspection of their error patterns on the experimental task revealed that the latter group tried to find a way to meet the constraints of the new task while counting from one end of the array to another. It is as if they took their regular counting experience as an instruction to continue to count in the conventional way.

The Gelman and Meck (1986) follow-up to Briars and Siegler (1984) supports this interpretation. If young children are asked to say whether a puppet's count is correct or not, they can fall into the trap of saying that novel but error-free trials of the kind generated in the doesn't-matter task are "wrong." This will happen more often than not if care is not taken to communicate that they should not mix up trials that violate conventions with those that violate principles. We did this by sharing with children the distinction between silly-but-OK ways to count, the regular way to count, and not-OK ways to count. Even if children interpret a setting correctly and know how to count, limits on procedural competence can cause errors (Smith, Greeno, & Vitolo, 1985). For example, to honor the constraints of the one-for-one count principles, one must have a way to keep separate tagged and to-be-tagged items. Limits on one's ability to generate plans with suitable sorting procedures can increase the tendency for one to loose track during a count. Likewise, children must learn to pace the rate at which they point to items in a display before pointing can help them to honor constraints against double-counting or skipping items (see below).

The Gallistel and Gelman (1992) model of nonverbal counting illustrates how there also can be systematic variability that follows from some aspects of the operation of the machinery in whose structure the implicit principles of the conceptual competence resides. In our model the preverbal counting mechanism generates mental magnitudes to represent numerosities; there is trial-to-trial variability in the magnitudes generated to represent one and the same set size; and this variability obeys Weber's law, that is, the standard deviation of the distribution increases in proportion to its mean. Given an additional systematic source of variability, increasing tendencies to loose track of what has been counted and what remains to be counted as set size increases, it is likely that the spread on the distributions as set size increases is even wider than predicted from the Weber law. This is important in understanding a potent within-task source of systematic variability in children's numerical performance, the effect of the set size.

A. Accounting for Variability in the Assessment of Numerical Competence

1. *Set Size Effects*

It is well established that variations in set size have a systematic effect on the tendency of infants and young children to respond correctly to the numerical information in a display. Infants' ability to use numerical information apparently is limited to set sizes of three to four. This fact has encouraged many to conclude that infants use perceptual mechanisms in lieu of mechanisms that embody implicit numerical principles. The favored perceptual mechanism is *subitizing,* an example of a process that is assumed to allow subjects to make discriminations between set sizes without any implicit or explicit understanding of numerical principles (e.g., Cooper, 1984; Cooper, Campbell, & Blevins, 1983; Fischer, 1991/92; Sophian, 1991/92; von Glaserfeld, 1982).

The preferences for a subitizing account of how infants respond to variations produced by different set sizes are tied to studies of adults' time for stating the number of dots in an array. Over the entire range of numerosities, the greater the numerosity, the longer the reaction time, but the first few increments in reaction time per additional dot in the display are smaller. (See Gallistel & Gelman, 1992; Mandler & Shebo, 1982, for reviews.) Because the slope of the reaction times functions in the small-number range ($N < 5$) is shallower than in the large-number range, it is commonly assumed that different processes underlie the responses to the small and large sets, subitizing and counting respectively. If one yokes infants' failures on larger sets to the assumption that a counting mechanism is needed for larger set sizes, it follows that infants are limited to the use of a subitizing process. This would allow infants to succeed with very small sets but make it impossible for them to succeed on larger sets. On the subitizing model, the ability to discriminate threeness from twoness is akin to the ability to discriminate "treeness" from "cowness"; unlike counting processes, this ability does not depend in any way on numerical principles.

There is no model for the kind of perceptual classification ability that is taken for granted by those who attribute infants' use of small-set information to subitizing. Still, we can ask whether this kind of account is consistent with the findings on numerical estimation in infants, children, and adults. The answer is, probably not. Whatever the classification process, classifications need not honor ordering principles. First, there is no reason for cowness to be mastered before treeness, or for the time needed to process "cow" to be systematically longer than the time needed to process "tree," and so on. Second, the perceptual hypothesis offers no account of why reaction times and error rates increase as a function of set size for both children (Chi & Klahr, 1975) and adults (Mandler & Shebo, 1982), no matter how small and how much they are practiced. Further, the postulated perceptual mechanism is peculiar in that it has to be indifferent to all sensory characteristics of the input. However, in order to recognize a cow, one must encounter cow-like stimuli, ones that look like cows, have cow parts, sound like cows, move like cows, and so on. To some extent the size, color, posture, and so on can vary, but overall shape or kinds and arrangements of parts cannot. In contrast, whatever the subitizing mechanism, it has to handle the fact that there are no restrictions on the degree to which inputs can vary in terms of size, color, orientation, and so forth. For geometric reasons there are some limits on the shapes that can be represented with a small number of distinct items, but nevertheless there is always more than one way to arrange sets of at least two items. Likewise, there are many common arrangements that can be imposed on larger sets.

If subitizing is a general perceptual process, then why should judgments of numerosity, even for small sets, serve as inputs for numerical reasoning processes? What is there about the perception of cowness or treeness that would lead one to ponder the effect of adding or subtracting items or to wonder whether one display has more (or fewer) items in it? Yet, we know that infants 12 months old and younger order different set sizes (Cooper, 1984; Starkey, 1992; Wynn, 1992a) and take into account surreptitious changes in the number in an expected set (Sophian & Adams, 1987; Starkey, 1992; Wynn, 1992a).

Whether the field should continue to favor an apprehension-like mechanism to account for the non- or pre-verbal numerical abilities of infants must await the development of a detailed model of this kind. Only then can it be determined whether it can handle the empirical and formal constraints covered here. For now, we prefer the Gallistel and Gelman (1991, 1992) nonverbal-counting model of performance in the subitizing range because it can handle the data from infant studies as well as the characteristics of the range of adult findings in the literature. It also is consistent with the Meck and Church (1983) model of the known abilities of animals to keep track of largish sets, in some cases as large as 50. An example of this ability in rats is shown in Fig. 1.

Platt and Johnson (1971) required rats to press a lever *N* times to arm a feeder that operated silently across blocks of trials as the *N* varied from 4 to 24. The *N* presses on the lever had to occur before the rat poked its head into the feeding alcove. If a rat failed to press the requisite number of times before he poked his head into the alcove, the counter was reset to zero. There are a number of features of the Platt and Johnson results that merit attention. First, the median number of presses corresponds to the required value of *N*. Second, note that the variance of the distribution for each *N* increases as does *N*. That is, the greater the numerosity to be represented, the more likely it was that the animals confused it with adjacent or nearly adjacent values of *N*. Third, there is even some overlap in the distributions around the mean for the smaller values. The Meck and Church (1983) nonverbal counting process, a process that the authors intended to have honor the Gelman and Gallistel (1978) counting principles, generates representations of numerosities as well as the characteristics of the Platt and Johnson data. Therefore, the data in Fig. 1 suggest an explanation for the failure of infants to discriminate between sets greater than three and four items.

If infants use a preverbal counting process like Meck and Church's, the fact that their discrimination is consistent for two versus three items but variable for three versus four items is as expected, given the Weber variance of the proposed nonverbal counting mechanisms. The increasing variance in the magnitudes that represent a given numerosity makes confusion of adjacent numerosities increasingly likely as set size increases. Put

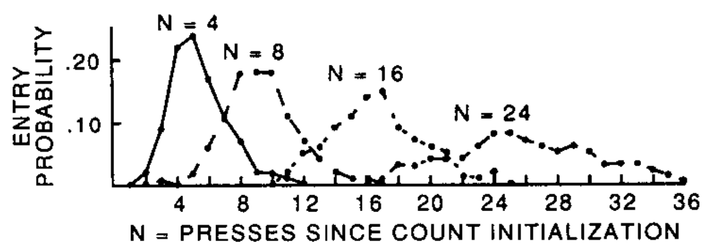

Fig. 1. Redrawing of the Platt and Johnson (1971) evidence that rats represent numerosities as large as 24—the probability of the rat breaking off to enter the food-delivery area as a function of *n*, the number of presses made since the initialization of the response counter, for various values of *N*, the required number of presses before the rat should enter the feeder. (The redrawing is from Fig. 1 of Gallistel & Gelman, 1992; with permission of the authors and publishers.)

differently, the animal experiments in effect measure the tendency to confuse one N with another as a function of the numerical distance between the Ns. Infants' numerical discrimination is scored as either correct or not. But this will not allow one to differentiate between errors that are due to increases in the inherent variability or an inability to use a preverbal counting process. It is therefore necessary to find ways to obtain from infants estimates of variability as a function of set size.

The Meck and Church (1983) mechanism for generating magnitudes to represent numerosities is a counting mechanism, not a pattern-perception mechanism; hence item kind is irrelevant, the representatives of numerosity are inherently ordered in accord with the set sizes and the representations of numerosity that can be related to the effects of addition and subtraction. The hypothesis that infants extract representations of numerosity by means of a preverbal magnitude-generating counting process dispenses with the need to postulate the rich abstraction and classification abilities that Piaget and others have taken to be the sine qua non of numerical concepts. First-order sense data are no longer primary. The primary elements are the objects qua discrete objects. The counting mechanism takes no account of their sensory attributes. Nor is there a requirement that a linguistic tag be used.

In the preverbal counting mechanism, mental magnitudes are used in place of count words. The process that generates these magnitudes is like the verbal counting process only in that it, too, honors the principles that define counting processes: the one-one assignment of numerons to the objects in the set, the stable ordering in the assignment of successive numerons, and the use of the final numeron assigned to represent the numerosity of the set. Given that variability is expected to increase as function of set size, both because of the nature of the magnitude-generating process and because of increasing information-processing demands of counting, the existing infant data cannot be used to reject the idea that infants use a preverbal counting process. Indeed, the data that are available are consistent with the Gallistel and Gelman (1991, 1992) model of infants' nonverbal counting competence.

2. *Variable Performance across Tasks*

I hold that nonverbal counting principles help beginning language learners to identify that set of speech units that can serve their learning of verbal count principles. There is some support for this proposal (Gelman, 1990; Shatz & Backscheider, 1991). Nevertheless, when 2- and young 3-year-olds are asked "how-many" questions, they do not do well. They either count without giving the cardinal answer, state the N without apparent counting, or even state some other N (Fuson, 1988; Wynn, 1990). Fuson takes this as reason to favor a model that has children acquiring separate component skills or rules before they achieve principled understanding of counting. Wynn uses these kinds of data to reject our proposals that nonverbal counting principles direct learning and that beginning language-users have implicit understanding of the verbal cardinal principle. She favors the hypothesis that verbal counting principles are induced as a function of counting-word experience with small sets. She defends her view by showing that comparably young children also have difficulty with two further verbal counting tasks, one that asks them to produce X items ("Could you give Big Bird two [three . . .] dinosaurs to play with?" p. 171) and one that asks them to match one of two sets to a numeral stated by the experimenter. My efforts to understand these reports and the consequent challenges to my theory led to the development of alternative ways to look at young children's verbal counting. They also took us back to some of our magic-task studies.

There is a cross-task contrast between the results for very young children from how-many, give-X, and other tasks and those from the "magic" task that I have used to uncover early understanding of addition and substraction as number-relevant operations. In work in progress, we have been exploring the factors that are responsible for the systematic effects of these tasks. We have also worked to develop new counting tasks that seem especially well suited for use with very young children. Below I summarize the findings from a new analysis that Gelman and Meck (1991) did with the protocols from a magic study that included 2½-year-olds. These data, in conjunction with those from a new number task, help illuminate the subtle roles of interpretative and procedural competence as factors in determining between-task variability in very young children.

a. The Magic Task Reconsidered Gelman & Meck (1991/92) reported a new analysis of the Bullock and Gelman (1977) "magic" experiment that was designed to determine whether 2½- to 5-year-old children could reason with and about numerical relations. The method embeds a magic show in a two-phase game. In the first phase, the experimenter hides a plate with one item under one can, and another plate with two items under a second can. Without mentioning the numerical values, the experimenter points to either the one-plate or the two-plate (the fewer and more conditions, respectively) and proclaims it "the winner." The displays are then covered with cans and shuffled. Children first guess which can might be hiding the winner, then look under the can they pick, and finally tell us whether they were correct. The trials of Phase 1 serve to establish expectancies for the displays. Phase 2 starts when the experimenter surreptitiously adds two items to each display. Children now have to decide which of the uncovered new values (three or four) is a "winner." Those who were initially rewarded for finding the one-plate now have to designate three the winner; those who were first rewarded for finding two now have to choose the four-plate to be scored as correct.

Bullock and Gelman reported that although 3- , 4- , and 5-year-olds succeeded in Phase 2 of the magic experiment, the youngest children did not, that is, the 2½-year-olds did not choose reliably on the basis of a common numerical relationship between Phase 1 and Phase 2. Bullock and Gelman's second experiment, dubbed the *control condition* by Gelman and Meck (1991/92) in their report of their new analyses of Phase-2 protocols, tested the possibility that the youngest children in what Meck and Gelman called the *regular condition* did not interpret the task as intended, that is, that they did not think to transfer the knowledge they acquired in Phase 1 when they encountered novel values in Phase 2. Two variants of the follow-up experiment provided hints to do this. In one, the initial displays were left in place with their covers on so as to suggest that there was something about these that was relevant to the way children should answer in Phase 2. In the other condition, the initial displays were left in an uncovered place in order to determine whether children's failures during the second phase occurred because they could not remember enough about the initial displays. Then, in Phase 2 of the control condition, the experimenter introduced a new game "like the one just played" and put two new covered displays on the table. A reliable number of 2½-year-olds now chose relationally in Phase 2, that is, correctly selected the four-item (*more*) or three-item (less) display. Bullock and Gelman concluded that limits on the youngest children's ability to think to transfer learning across conditions were responsible for their poor performances in the regular condition. Because it is unlikely that such limits are domain-specific, they should be considered whenever one interprets how very young children do on transfer tasks. In any case, since the hints to transfer worked for Bullock and Gelman, it was possible for us to return to the Phase-2 protocols to determine whether they contained evidence that these young children applied the how-to-count principles.

After children first answered the Phase 2 winner questions for the altered display(s), Bullock and Gelman continued with an interview to determine whether they knew how many items were present during both Phases 1 and 2, how many were added or removed, and so on. Because these questions were asked for different values that were either physically present or to-be-remembered, this is a setting in which children might interpret correctly how-many questions. Indeed, because children had reason to compare different set sizes (those in front of them as well as those from the first phase), they also had reason to count more than once. Such efforts generated the data for the Gelman and Meck (1991/92) conclusion that these children did use verbal counting principles, despite the fact that the sets represented very small *N*'s.

The details that influenced Wynn's conclusions led us to focus on whether children's use of number in Bullock and Gelman reflected an ability to count *and* to offer the cardinal value for set sizes of three or smaller. Wynn acknowledges that young children will count small sets when asked to do so; her claim is that they cannot relate these counts to a verbal cardinal principle. Behaviorally, this translates into a prediction that they will not count and offer the cardinal value of even small sets when asked about cardinal number. Therefore, in one of our analyses of the transcripts, we only scored subjects as having given both a cardinal and a counting response if they either (1) did so spontaneously, or (2) offered two kinds of answers to the same question ("How many?" or "Can you count . . . ?"). Pairs of trials in which children used the cardinal value on their own but counted only after we asked a question were not scored for this analysis. Nevertheless, more than 60% of the children gave evidence of using the cardinal count principle under these conditions, a percentage that is reliably greater than expected on the basis of Wynn's data.

In sum, the reanalysis of the Bullock and Gelman experiments provides some evidence that 2½-year-olds can apply their implicit understanding of counting principles—at least given the conditions like those that hold in the magic experiment. What is it about the magic task that brings out this competence? My answer is that there was a reason for the children to count and relate the result to the cardinal values they actually saw as opposed to those they had expected, for they encountered unexpected changes in set size. How-many questions do not share these conversational conditions. Because we are not supposed to repeat what is known, the child who shares knowledge of the verbal counting principle with an interviewer might assume that it is sufficient to provide either the cardinal value or the count. When a speaker counts aloud, there is no need to repeat the last count to signal its status as the cardinal value of the set; the auditor can hear the last count word. It would be a violation of conversation rules to signal something so obvious to an adult listener. Conversely, the statement of the cardinal numerosity may be taken to imply that there was a count. We checked our intuitive beliefs about the force of these conversational rules in this context by running adults in a how-many study. When we asked undergraduates a how-many question about 18 blocks, all of them counted but only one bothered to repeat the last count word said. Repeats of the question elicited puzzlement, some recounting, and so forth, the kinds of responses that index the fact that we violated the conversational rule *don't ask about what is given*—in this case the numerosity implied by the count. The implication is clear: To repeat the how-many question is to run the risk of confusing subjects as well as confounding assessments of interpretative and conceptual competence.

The reader might object that what is so for adults may not be so for beginning language users. It is therefore necessary to find ways to obtain better evidence that very young children can and do use verbal counting principles—even on small set sizes. A new study with 2½- and young 3-year-olds accomplishes this. It also allows us to tease apart the contributions of interpretative, procedural and conceptual competence.

b. The What's on This Card? Study We spent a great deal of time watching and comparing beginning talkers' reactions to a variety of number-relevant conditions, as opposed to non-numerical ones. (This study was done in collaboration with Betty Meck and Lisa Kaufman.) We were repeatedly impressed with how much their attention wandered when we tried to keep them focused on how-many questions or other instructions that started out asking about numbers. In contrast, if we asked them to label objects or kinds of displays, they were more likely to stay on task. This contrast led us to a new number task that takes advantage of these very young children's interest in labels. In this new task, the experimenter starts by showing a child a card with one sticker on it, for example, a bee, and asks, "What's on this card?" Children invariably say "a bee," at which point the adult responds, "That's right, *one* bee." Then she shows the child a card with two bees on it and asks, "What's on this card?" Invariably the eight children in our pilot study next used a cardinal number in their answer and said, "Two bees." Given additional trials with more items of the same kind, the children started to count to answer that they were looking at an *N*-bee card. As the experimenter continued to introduce cards with yet more bees, some children adopted yet another answering style. Now they counted and gave the cardinal value of the display but dropped any reference to the item type. When we finally shifted to cards with a new kind of item, some children once again gave both the kind and cardinal value that went with their count. This answering pattern fits well with conversational rules (Siegal, 1991) and suggests that children even this young know to leave unstated information that is old and already given (Clark & Brennan, 1991). Although there is no reason to keep labeling an object that has already been talked about by both conversational partners, one should provide the label when the subject is queried about a new item.

We have gone on to a more extensive study with the what's-on-this-card task. The latter followed a pretest session that assessed how well children in three different age groups could count and answer how-many questions for set sizes from 2 to 10. The groups' median ages were 2 years, 7 months (range = 2 years, 6 months–2 years, 10 months); 3 years, 2 months (range = 2 years, 11 months–3 years, 2 months); and 3 years, 4 months (range = 3 years, 3 months–3 years, 5 months), respectively. Below these the groupings are referred to as the 2½-year-old, youngest 3-year-old, and oldest 3-year-old groups.

To start the counting pretest, we placed two or three figures on a table and asked how many "people" were on the table. Since there were to be repeated trials, variants of the question (e.g., "How many are on the table now?"; "How many are there?") were allowed. The question "How many?" prompted both counting and cardinality responses. If necessary, the experimenter assisted the subject by either pointing to the items while the child provided the labels or by suggesting that the child begin the count sequence with "one." Then the experimenter again asked, "How many?" Additional trials (anywhere from two to seven trials per set size) were allowed in order to maximize the chance of the subjects counting a given set.

The what's-on-this-card session(s) followed on another day. Our design of this task called for the children to respond to what's-on-this-card trials for *N*'s of at least 2 through 5 and, as many as possible for those planned for set sizes six and seven. Materials for each set size were homogeneous collections of seven kinds of stickers (cats, frogs, candy canes, apples, bees, hearts, shoes) glued onto cards in either in one or two rows. The order of item type and set size within the subset of cards with the same items varied save for two exceptions: Every child started with the one-item and two-item cards for each new item kind. If counting difficulties were not evident during pretest trials, the child usually was presented all values (one–seven) of a given kind of sticker. In any case, we planned to test all children on all cards with values of two through five and told experimenters to persist with the study at least this far. If a child seemed to tire or wanted to stop, the interview was to continue on yet a third day. As in the counting assessment phase, although the interviewer could point for children during the counting phase, she could not help on a cardinal trial and the pointing help could be offered only after the child tried the trial on his or her own.

Different initial responses to the what-kind question led to different subsequent inquiries. For example, some children responded to, "What kind of card is that?" by simply stating the number of items without counting. In this case, the child was asked one or more probe questions, for example, "How do you know that's_____?"; "Can you check to make sure?"; "Can you show me?", and so on. Similarly, children who counted but did not repeat the cardinal value (e.g., "one, two, three bees," as opposed to "one, two, three; three bees") were asked, "So, what is on the card? How many X's is that? or How many?" and so on.

Children were strikingly more likely to apply all of the one-one, stable and verbal cardinal count principles on the what's-on-this-card task than they were on the pretest where we asked how-many questions. (See Table I.) Comparisons of a child's best trial during the pretest as opposed to the same child's best trial during the experimental session favored the what's-on-this-card task. So did a comparison of the number of correct trials per set size (Sign test, $p < .001$). As in our counting assessment pretest, the what's-on-this-card task brought out very young children's ability to apply the verbal one-one, stable order and cardinal count principles. For example Subject 2 (2 years, 7 months) answered with "Three bees; . . . 1, 2, 3 bees," and Subject 13 (3 years, 0 months) answered "1, 2, 3, 4, 5 . . . 5." For the pretest and experimental sessions, to be credited with knowledge of how to apply the verbal one-one, stable, and cardinal count principles, children had to (1) count and state the cardinal value correctly on at least half of their trials for a given set size; or (2) achieve a count of $N \pm 1$ and to pair it with the last tag of their $N \pm 1$ count. Additionally, if, on their first trials with set sizes of two and three, children simply stated the cardinal value, we required evidence that they were not simply "labeling" the set size before we credited them the cardinal principle. Such evidence could involve successful counts that were combined with correct statements of the cardinal value for larger set size or a show of N fingers at the same time that a cardinal value was provided on the smallest set sizes. Experimenter-assisted counts (that is, ones on which the experimenter pointed) were scored as having honored the one-one and stable order principles if children correctly used the requisite N conventional tags (or ones from their own idiosyncratic list). To receive credit also for use of the cardinal principle, children had to tell us the cardinal number (either N or $N \pm 1$ if a count was of this form) *on their own.*

A second scoring of children's success levels followed the same rules as those above with one exception. Now assisted trials were excluded from the summary counts used to determine whether a child met the at-least-50%-correct criterion. An even more stringent rule was adopted for a third scoring of the data. If, for a given N, there were any assisted trials, that child was scored as having failed to apply the count principles on that set size. Figure 2 shows the results of the most lenient analysis as a function of age and set size. The results from the two alternative ways of scoring success levels are shown in the bar graphs in Fig. 3. These results are presented with bars for the more lenient analyses shown in Fig. 2.

Figure 2 summarizes performance levels when correct assisted trials were counted in the at-least-50%-correct criterion for crediting a child with

TABLE I

A COMPARISON OF CHILDREN'S ABILITY TO USE ONE-ONE, STABLE, AND CARDINAL COUNT PRINCIPLES TRIALS ON HOW-MANY? AND THE WHAT'S-ON-THIS-CARD? COUNTING TASKS[a]

Age group and set size	Counting task: How many[b]		Counting task: What's on this card	
	Assisted	Unassisted	Assisted	Unassisted
2½ yr				
2	30	20	90	90
3	30	20	70	70
4	10	10	70	40
5	30	20	70	30
6	10	0	30	10
7 or more[c]	10	10	30	10
Youngest 3's (2;11–3;2)				
2	90	90	100	100
3	80	80	100	90
4	60	40	100	80
5	60	50	90	90
6	30	30	80	80
7 or more	50	60	80	80
Youngish 3's (3;3–3;5)				
2	80	80	100	80
3	80	80	80	80
4	70	70	80	80
5	70	70	80	80
6	60	50	90	80
7 or more	60	50	80	60

[a] Summaries are based on best trials.

[b] During the how-many counting pretest, subjects were asked how many "people" there were on the table. Because there were to be repeated trials, variants of the question, e.g., "How many are on the table now?"; "How many are there?" were allowed. Set sizes could range from 2 to 10. If necessary, the experimenter assisted the subject by either pointing to the items while the child provided the labels or suggesting that the child begin the count sequence with "one." Then the experimenter again asked "how many?" Additional trials (anywhere from two to seven trials per set size) were allowed in order to maximize the chance of subjects counting a given set.

[c] Children received credit in this case if they used all three principles correctly on at least one of their trials with a set size of seven or more.

all three verbal how-to-count principles. As compared to the findings for the how-many task in the literature (e.g., Wynn, 1990) and our pretest, the what's-on-this-card task yielded clearer evidence that very young children can use verbal counting principles. Save for our "oldish" 3-year-old

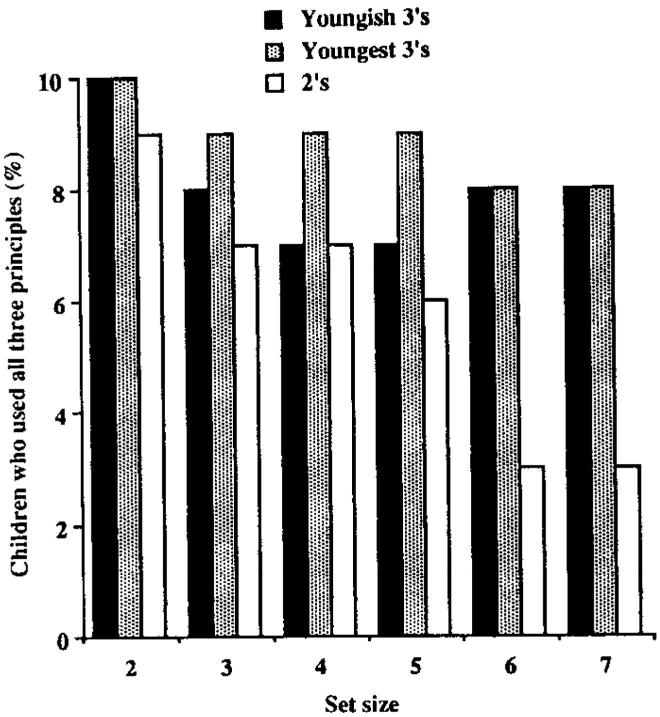

Fig. 2. The percentage of children in each age group in the what's-on-this-card task who used all three verbal how-to-count principles on different set sizes.

group, the children are below the average age for which we should see such results according to other investigators (e.g., Fuson, 1988: Wynn, 1990). Even when tested on set sizes of five, six out of ten 2-year-olds gave us some evidence of knowing how and when to use verbal instantiations of the counting principles in these ranges. And both of the other age groups provided even more substantial evidence to this effect.

If we apply more stringent criteria, we reduce the set size range for which we might attribute this competence. Figure 3 reveals a notable Age Group × Set Size × Scoring Criterion interaction. The younger the children, the more the effect of the scoring criteria. This is due to the fact that it was primarily the youngest children who improved when the experimenter pointed to the items being counted, especially as set sizes increased. Should this dependence on an adult helping them be counted against them? I think not. First, a considerable majority of the two youngest groups passed the most stringent criterion on set sizes of two and three, that is, when they were least likely to have an adult point for them. Even this extent of the ability to both count and apply the related cardinal value

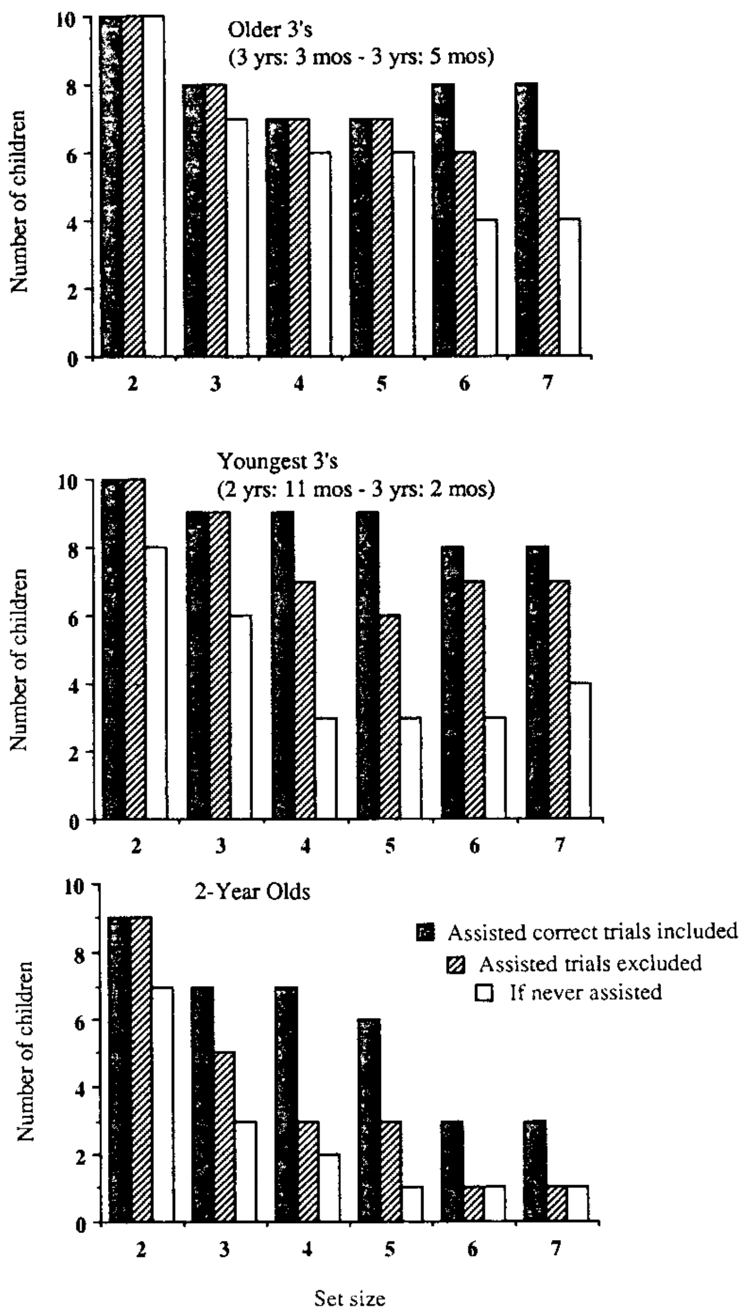

Fig. 3. Evidence that children in each group in the what's-on-this-card task used all three verbal how-to-count principles on different set sizes as a function of scoring criteria.

exceeds expectations, given results from standard tasks. Therefore, such evidence alone is consistent with an attribution of conceptual competence.

Second, Gelman and Greeno (1989) have noted that, unlike the ability to repeat the last tag of a count, pointing behaviors are not uniquely linked with the counting principles. They become counting linked because they help one honor counting principles, much as they help one sort items. For this reason, we propose that the data fit well with our idea that young children's conceptual competence can be masked by limits on their procedural competence. We never offered children clues regarding the cardinal value of the set, and even the youngest children performed well without assistance on small sets. Because they are shaky counters when left on their own with larger set sizes, the odds are that they have not had much experience in this range. This makes it unlikely that they were simply mimicking others, especially when adults may not offer such models (Gelman, Massey & McManus, 1991). Conversational rules inhibit their pointing out the obvious, as we showed in our how-many study with adults (Gelman & Meck, 1991/92).

When we watched the videotapes from this new experiment, it seemed that an adult's pointing worked in part because it kept the youngest children more task-oriented. Thus younger children's better performance on assisted trials also could have been related to matters that bear on interpretative competence. Because it is unlikely that the youngest children would have had occasion to learn to perform such tasks on their own (Siegal, 1991), the assisted trials could have served to create a setting of the social kind young children expect when they are to show others what they know (Rogoff, 1990; Siegal, 1991; Vygotsky, 1978). Analyses of the success children had with different kinds of questions provide evidence for our belief that very young children are sensitive to these kinds of interpretative matters.

As indicated above, we were prepared to ask children to justify their answer with different kinds of probe questions. These included "How do you know?"; "Can you show me why (it's 4)?"; "Show me"; "Can you (let's) check." Because beginning language learners are far from fluent with mental verbs (Shatz, Wellman & Silber, 1983), but very good with verbal requests for action (Shatz, 1978), the fact that we varied these formats gave us a direct assessment of the extent to which interpretative competence played a role in children's ability to reveal understanding of the counting principles and the relation between the verbal one-one and stable principles, on the one hand, and the cardinal verbal principle, on the other hand. As shown in Table II, the pattern of answering is consistent with the proposal that interpretative competence interacts with young children's conceptual competence for counting. All age groups of children

TABLE II

EFFECT OF QUESTION TYPE ON CHILDREN'S ABILITY TO ANSWER

	Kind of question					
	Show			How know		
Age group	Mean no. questions asked[a]	Number answer (%)	Irrelevant or no answer (%)	Mean no. questions asked[a]	Number answer (%)	Irrelevant or no answer (%)
2½	6 (10)	78	22	3 (7)	50	50
Youngest 3's	5 (10)	88	12	3.3 (9)	33	66
Youngish 3's	3.3 (6)	95	5	3 (10)	44	54

[a] Number in parentheses is the N of the 10 subjects/age group who contributed to the mean number of times this question was asked.

were better able to deal with show as opposed to know-how questions. In fact, the rate at which the children answered questions like "How do you know that it is five?" is far from impressive—no matter what their age group. Since this is a commonly asked question in how-many studies, here is yet another source of interpretative competence that confounds assessments of young children's understanding of verbal count principles.

Overall then the evidence from the what's-on-this-card study helps develop our ideas about the need to consider the sources of variable performance when reaching conclusions about a given kind of competence. The role of interpretative competence is surely of significance when subjects are beginning language users. Since there is good reason to say that they will apply conversation rules variably, especially ones that suit experimental settings, this alone can confound the assessment of conceptual competence. Similarly, the young have problems coordinating actions. This too can influence how competent a performance they will render. Rather than insisting on the use of tasks that require such skills, we should make every effort to control these problems or to get around them. For it is certain that the younger the child, the less her ability to output the requisite plans of action. From this point of view, complaints about studies of early competence that minimize action requirements (e.g., Fischer & Biddell, 1991) should be redirected to finding out whether the young recognize the relevance of suitable actions, even if they themselves have limited or no ability to execute them. Our pointing during a counting trial can be viewed this way, as can studies on preschool children's abilities to choose between correct but novel and incorrect counting sequences that are generated by a puppet (Gelman & Meck, 1986). The conditions of the latter task resemble language use studies that ask individuals to discriminate between grammatical and ungrammatical utterances. To do so requires implicit understanding of the rules that characterize the class of acceptable utterances in one's language. Similarly, a child cannot not succeed at discriminating "errorful" from unusual, but acceptable, count trials without implicit principled knowledge of counting. Parallel data pertaining to the question of whether young infants possess an object can be used to buttress the idea that limits on the ability to generate and execute action plans can mask one's conceptual competence in a domain.

B. Variability across Assessments of Infants' Concept of an Object

If the planning system needed to generate a competent plan of action is limited or not even developed, then the risk is high that a child will fail no matter how much she knows about the content domain at hand. Piaget's tests of the infant's understanding of the object concept all require the deployment of a competent plan of action. Yet, there is every reason to presume that infants' abilities to generate coordinated actions go through a protracted developmental course. Might it be that the conclusions Piaget reached about the development of infants' understanding of object permanence are better thought of as developments in infants' ability to assemble suitable plans and related action sequences? Fortunately, there are studies that bear on this question and the interpretation of the A-not-B error that infants make when they watch as an object is first hidden at one place (A) and then visibly moved to another hiding place (B). There is a point in development when an infant who can retrieve an object that is hidden at position A fails to retrieve the same object that they have just seen being displaced and hidden in position B. Within Piagetian theory, their tendency to look for the object at the first hiding place (A), as opposed to the second one (B), is taken as evidence that they do not understand that objects remain intact as they are visibly displaced through space. There is another account of the A-not-B error pattern.

To start, we know that the information-processing demands of the task vary as a function of development. Diamond (1985) finds clear effects of varying how long an experimenter waits to move the object from Position A to Position B, as well as of when the infant is allowed to reach for the hidden object. If the delay between the end of the hiding phase and the beginning of the retrieval phase is less than 2 sec, $7\frac{1}{2}$-to-8-month-old infants do not make the A-not-B error; instead they do look for the object in its second hiding place. If the interval exceeds this time, they make the typical error of returning to the original hiding well. By 9 months of age, infants can tolerate delays up to 5 s, and by 12 months up to 10 s. Bjork and Cummins (1984) used a five-well hiding arrangement to show that infants who erred had a clear tendency to choose an item that was near the first hiding place.

But it is not just the limits on memory that hinder the immature infant's ability to reach for the correct item. Diamond (1991) details a number of variables that limit the 5- to 7-month-old infant's ability to assemble and/or produce a competent plan of action. For example, younger infants have trouble inhibiting reflexes that are elicited if they accidentally touch the side of an object that happens to be near the one they have to get. Additionally, they have trouble meeting the requirement of putting two separate acts together. Baillargeon, Graber and Black (1990) also make it clear that there are action-specific limits, that is, limits that are not tied to knowledge about object permanence, that can mask the infant's knowledge of objects. These authors show that 5½-month-old infants know the difference between actions that can and cannot support the retrieval of an object that is hidden behind a screen. In one experiment infants watched an object being retrieved under a possible and impossible action condition. To set the stage for the relevant part of the experiment, it was first necessary to have infants watch one of two kinds of hiding events. The first one involved the hand of the experimenter placing a toy teddy bear on a table, the subsequent movement of a screen in front the bear, and then a hand reaching behind the screen and pulling out the teddy bear, a perfectly possible event. In the second one, a toy teddy bear was placed on a table with a see-through cup over it, and then a screen was moved to occlude the display. Then, while the screen was in place, the infant again watched as someone's hand reached behind the screen and pulled out the teddy bear, an impossible outcome. Infants responded appropriately to these two different events. In subsequent conditions they also were able to indicate appreciation of the fact that a more complex action sequence was required for retrieval of the teddy bear in the second condition, the one in which the cup would have to be removed before the bear.

I share with Baillargeon and Diamond the view that at least part of the difference between outcomes in studies that use habituation of looking as opposed to reaching responses to evaluate whether infants believe in object permanence, has to be attributed to general limits on the ability to produce suitable action sequences. Diamond's demonstration of comparable action problems in delayed-matching-to-sample and novel-object-reaching tasks adds weight to the conclusion that there is much to learn about the development of action planning, and therefore procedural competence, during the first few years of life. Together such studies also serve to answer those who want to dismiss findings by Baillargeon and Spelke on the grounds that they come from only one task, the habituation of looking paradigms. Even if this were true, it is not a fair criticism, because infants in habituation and visual-preference studies have been tested under a wide range of stimulus conditions, including ones in which balls fall to the floor, trains go down tracks, blocks and toys are hidden, objects are felt but not seen, and so forth. The studies that now show infants able to discriminate between relevant and irrelevant retrieval acts and to pass the Piagetian task if they are tested under the right conditions help corroborate the conclusions that were based on habituation methods. Additionally, they give me further reason to argue that part of what develops is an ability to relate conceptual competence to procedural competence.

My position leads me to conclude that there is a sense in which Piaget was right to focus on the role of action in thought. However, I differ on how to interpret the development of more and more coordinated actions. For Piaget, thought and action are almost one and the same. For me they are different. Infants start life with immature and limited abilities even to perform, let alone put together, those acts into a competent plan that can satisfy conceptual competence. It is wise therefore to use assessment techniques that minimize demands on the procedural side if the question of interest is what conceptual competencies infants might have. It is better still to attempt to find more than one such method, or at least to vary the test materials across experiments. As these variations increase, so does one's confidence in the attribution of the competence in question. It is because there are many different studies of very young infants' ability to treat objects as permanent that we are now able to show that there are systematic sources of variability in Piaget's task that are not due to the extensive kinds of conceptual limits that Piaget would want us to place on infants.

IV. Conclusions

In case it is not clear by now, my position is not that our young come to the world with full conceptual competence in a domain. The choice of a skeleton as metaphor is meant to make this obvious. My claim is that this learning is much aided by the presence of structures of mind, no matter how limited these might be, because whenever it is possible for an individual to use his or her own organizations to interpret and store new inputs, learning is facilitated. Conversely, learning is much harder when one has no ideas as to how to interpret or organize novel inputs.

First principles can aid learning whenever the inputs do or can share the structure of the domain. If the structure of the inputs cannot be mapped onto some mental structure, no matter how limited the structure might be, then learning is likely to be at risk, because one is confronted with the conceptual analogue of having to get to the middle of a lake without a row boat. There is nothing akin to a skeletal structure to support correct interpretations of inputs and/or a coherent storage of them. Somehow we have to build up a structure at the same time that we accrue bits of knowledge that can feed the development of the structure that is being assembled. Elsewhere I take on the question of how schoolchildren start to succeed at this extremely difficult task. My focus in Gelman (1991) is on an account of how they get beyond the belief that numbers are just those things we get when we count things. To understand that fractions are numbers, one has to understand that a fraction is one cardinal number divided by another. However, the counting principles cannot support this notion. In the absence of a relevant mental structure, the task then becomes one of finding mental steppingstones. One source for the development of such "stones" is the repository that results from the mapping of a bit of a system that is already understood to a bit of a conceptual system that we might come to know. I intentionally say "might come to know" because it is clear that learning about new domains or new ways to construe knowledge within a domain is far from easy. Children might readily agree that they can count 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$ . . . but insist that there are no numbers between 0 and 1 (Gelman, 1991). To accept the latter is to give up the idea that numbers always involve counting things. Put differently, it is not easy to build conceptual competence in the absence of an existing knowledge base or at least a skeletal set of principles.

In sum, performances reveal understandings that are the result of a collaboration between several different competencies. There is conceptual competence, which is the ability to generate a relevant representation and to apply appropriate principles of reasoning to that representation. An example of this kind of competence is the ability to generate a representation of an object and to apply the principle that two different objects may not both occupy the same part of space (the principle that makes the full backward rotation of the screen impossible). Another example is the ability to generate representations of numerosity and to recognize the ordering of these representations and the effects of adding and subtracting them. By experimenting with different tasks, it has been possible to obtain strong evidence that these conceptual competencies are present in infancy. As we vary the tasks within a conceptual domain, we bring into play various interpretive and planning competencies. Our understanding of the underlying conceptual competencies has enabled us to begin to tease out the contribution made by these other competencies to the lack of success or to the failure that we observe across tasks that are supposed to tap the same conceptual competencies. Our understanding of the characteristics of the preverbal counting process has also allowed us to model the child's inability to make reliable discriminations between adjacent numerosities outside the range of three or four items. All of these results strengthen the case for a constructivist–rationalist model of eary cognitive development. The domain-specific principles that underlie conceptual competence are present at a very early age. They guide the active assimilation of the relevant aspects of experience. These aspects are not the first-order sense data or elementary action patterns that have been assumed to be the foundation of cognitive development. Rather, they are the aspects that satisfy the structural constraints imposed by the domain-specific principles that guide development. Were there no first principles, our young would have as much difficulty acquiring their first understandings as do individuals who come to master understandings that cannot build off first principles or existing knowledge schemata.

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