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4 Basic Numerical Abilities

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Typically, definitions of human intelligence include some reference to mathematical abilities. Most IQ tests have questions requiring some knowledge of mathematics or arithmetic. Piaget highlighted the role of mathematical-logical knowledge in the development of concrete and formal operations. And, people who are able to do mathematics are thought to be bright—at least in Western cultures. Why the pervasiveness of the assumption that mathematical abilities contribute to intelligence? I think it follows from a general view that "abstract" thinking abilities are fundamentally involved in "intelligent" thinking. By definition, mathematical ideas are abstract. This is even true for our ideas about natural numbers—they are *not* "out there" in the real world waiting to be noticed. Hence, by implication, mathematical ideas represent an advanced level of intellectual ability.

I believe the assumption that mathematical thought is both abstract and intelligent has contributed to our willingness to presume that the concept of number and arithmetic abilities are relatively late on the developmental scale. Be it Piaget (1967), Vygotsky (1962), or Bruner, Olver, and Greenfield (1966) to whom we appeal, the view is that abstract concepts are not available to preschoolers. Against this theoretical backdrop, it is easy to accept the argument that arithmetic concepts likewise are not available to preschoolers. It is common to read that preschoolers count by rote with no understanding of the counting procedure (Piaget, 1952; Saxe, 1979a, 1979b), let alone an accurate concept of number. Piaget's finding (1952) that preschoolers fail the conservation of number task served to buttress the view that number concepts are late in developing. However reasonable such assumptions might seem, they are probably wrong. Recent evidence points to the conclusion that certain mathematical abilities are present during the preschool years. Indeed, it seems that the ability to count and do simple arithmetic problems may be as natural as the ability to speak a language.

In this chapter, I review the findings on the nature of early counting abilities—findings which support the view that very young children have implicit knowledge of counting principles. Next, I consider how implicit knowledge of counting might develop into explicit knowledge, including a beginning understanding of infinity. Then I take up the functions that can be served with this knowledge of the principles. I then marshal the evidence that counting and related abilities involve natural, universal cognitive abilities. I end with a discussion of the limitations of this conclusion.

A DEFINITION OF COUNTING

In order to assess the young child's ability to count, it helps to have a definition of counting as a yardstick against which to compare performance levels. According to Gelman and Gallistel (1978), counting involves the coordinated application of five principles. This list of principles was derived from a consideration of formal definitions, existing psychological models of counting (e.g. Klahr & Wallace, 1976; Schaeffer, Eggleston, & Scott, 1974), and an initial analysis of what looked like counting sequences generated by preschool children. The principles are: (1) the one-one principle; (2) the stable-order principle; (3) the cardinal principle; (4) the abstraction principle; and (5) the order-irrelevance principle.

As Gelman and Gallistel (1978) point out, every known counting model assumes the use of the one-one principle, which involves ticking off the items in an array with distinct tags so that one and only one tag is used for each item in the array. In following this principle, an individual has to coordinate two component processes: *partitioning* and *tagging*. Partitioning involves the step by step maintenance of two categories of items—the to-be-counted and the already-counted categories. Items must be moved (physically or mentally) from the latter category to the former. The partitioning process must be coordinated with the tagging process, which involves the summoning up of distinct tags, one at a time. These are typically the count words, but they need not be. As long as the set of tags is distinct and different from the names of attributes of the to-be-counted items, it can serve the tagging function.

Although counting must involve the one-one principle, the use of this principle by itself does not constitute counting. At the very least, the one-one principle must be applied in coordination with the stable-order principle. That is, the tags used in a count must be arranged or chosen in a stable, repeatable order. The principle requires the availability of a stable list that is at least as long as the to-be-counted number of items requires it to be.

The one-one and stable-order principles involve the selection and application of tags to the items in a set. The cardinal principle captures the fact that the *final*

tag in a count sequence has a special status. It, unlike any other tags, represents the number of items in a set. This principle presupposes the first two principles and the ability to pull out the last tag in a sequence for use in indexing the cardinal value represented in an array.

The three foregoing principles together constitute the *how-to-count* procedure of the Gelman and Gallistel model. The fourth principle—the abstraction principle—captures the fact that the counting procedure can be applied to any collection of real and imagined objects. Although not a common practice, we could in principle collect together for a count such disparate items as the letters of the alphabet, all pieces of furniture in a room, and the number of minds in that room. For adults, at least, any set of any combination of discrete things can be counted. This is so obvious that one might question its elevation to the status of a principle. The reason comes from the developmental literature. Many have maintained that young children severely restrict the definition of countables. Ginsburg (1975) wrote that early counting and the concept of number are: "tied to particular concrete contexts, geometric arrangements, activities, people, etc. It is a long time before the young child treats number as abstract [p. 60]." Gast (1957) advanced a similar view and, on the basis of his counting experiments, concluded that only children 7 years of age or older have a fully abstract conception of what can be counted.

Gast shared a widespread view of how a child comes to recognize that any kinds of objects can be put together for a count. To do this, the child must recognize that all objects can be assigned to the common category *things*. By one account of development, this ability is very abstract. This follows from a common theoretical argument about the nature of classification abilities, namely, that they proceed from being extremely concrete to very abstract (cf. Bruner et al., 1966; Vygotsky, 1962; Werner, 1957). The idea is that the ability to classify objects as things is the result of being able to form an extremely elaborate hierarchy of subcategories wherein the superordinate is *thing*. Hence, the view is that children slowly develop a more and more abstract conception of *thingness*. If so, what may be obvious to the adult may not be obvious to the child.

Should it turn out that young children are relatively indifferent to the definition of countables, it is not necessary to conclude that they make use of a complex scheme for constructing hierarchies. One can view the ability to classify the world as things and nonthings as a close derivative of the ability to separate figures from grounds. Following this interpretation, the categorization of things as opposed to nonthings may well be among the earliest mental classifications. Hence, should the data reveal an early ability to classify heterogeneous items together for counting, we have an alternative to the conclusion that this ability implies the use of a hierarchical classification scheme.

The order-irrelevance principle captures a crucial fact about the adult's knowledge of counting. In many respects, it does not matter which tag is assigned to which object. Any object can be tagged with any of the appropriate count words (i.e., those in the stable-order list). Given a linear array showing pictures of a star, a circle, a triangle, a nonsense shape, and so on, it makes no difference which item is tagged as *one*. Furthermore, it is perfectly acceptable to designate the star as *one* on trial *a* and the triangle as *one* on trial *b*. As long as each item is uniquely tagged and the stable-order principle is honored, the order in which items are tagged is irrelevant. The child who recognizes this fact can determine that the cardinal number of a set is the same, no matter which items are tagged *one, two*, etc.

Ginsburg (1977) cites anecdotes of young children who seem to believe that a given number word becomes attached to a given object. Piaget (1952) makes much of a child who was surprised to discover that a set of 10 objects was still a set of 10 objects even if a count was started at a different point in the array. If preschoolers do not recognize that the order in which items are tagged is irrelevant, it would be difficult to conclude that they understand the role of counting in the quantification of a display. Counting represents a set of procedures for generating a numerical representation, a representation that is not a direct perception of things "out there" in the world. A child who insists that the first object is called *one* fails to recognize that much about counting is arbitrary.

Children who honor all the counting principles clearly know how to count. But what about children who make errors? It depends on what kind of errors children make as to whether we grant them implicit knowledge of any one principle or any combination of principles.

EVIDENCE THAT YOUNG CHILDREN DO "KNOW" HOW TO COUNT

A Caveat

It is important to recognize the distinction between an implicit and explicit understanding of principles or rules. Young children are granted implicit knowledge of linquistic structures well before they are granted explicit knowledge (cf. de Villiers & de Villiers, 1972; Gleitman, Gleitman, & Shipley, 1972), which is often characterized as metalinguistic. As we see later, a similar distinction can be made concerning counting principles.

Evidence for Implicit Knowledge of the Three How-To-Count Principles

A child is granted implicit knowledge of the rules of a language on the basis of at least two kinds of data—the systematic production of sentences of given complexity (e.g., Brown, 1973) and overgeneralization errors like *mouses, footses, unthirsty*, etc. that can only be explained by reference to the availability of rules (e.g., Berko, 1958; Brown, 1973; Clark & Clark, 1977). In 1978, Gelman and

Gallistel provided comparable evidence that even some 2-year-olds honor the three how-to-count principles and that many 3-year-olds honor all five counting principles.

The 2½-year-olds. Perhaps the most compelling evidence that some 2½-year-olds have implicit knowledge of the how-to-count principles is the use of what Gelman and Gallistel call idiosyncratic lists and Fuson (e.g., Fuson & Richards, 1979) calls nonstandard lists. These appear in very young children even when they count larger set sizes. Although the lists are nonstandard, they are nevertheless used systematically. Thus, for example, a 2½-year-old child might say "2-6" when counting a two-item array and "2-6-10" when counting a three-item array (the one-one principle). The same child will use his or her own list over and over again (the stable-order principle) and, when asked how many items are present, repeat the last tag in the list (the cardinal principle). Gelman and Gallistel note that the 2½-year-old who uses an idiosyncratic list is better able to use it in the same order over count trials than one who works with the conventional list. This observation fits with the fact that subjects who impose their own organization on material are better able to recall it (e.g., Mandler & Pearlstone, 1966). The latter argument presupposes that the child is honoring some rule or organization; the stable-order principle is as good a candidate as any we can think of.

The 2½-year-old also reveals some—albeit not a perfect—ability to honor the one-one and cardinal principles. When asked to count the number of items in an array, there is a systematic tendency to use more tags for set sizes 4 and 5 than for set sizes 2 and 3 (Gelman & Gallistel, 1978). Although the numerical relationship between the number of tags and objects is imprecise, it is far from random. Hence, Gelman and Gallistel suggest that 2½-year-olds recognize that counting involves assigning tags to items in an array. They also report that 50% of the 2-year-olds who participated in a counting experiment were able to identify the cardinal numerosity of a two-item array, and all of the same children could count the same number of items when asked. Only 25% of the children could likewise count and then identify the cardinal value of three-item arrays. Gelman and Gallistel conclude that the tendency of very young children to apply the cardinal principle is weaker than the tendency to apply either the stable-order or one-one principle, an observation that finds support in the analysis of the counting abilities of 3- to 5-year-old children.

The Older Preschoolers. The main evidence for the Gelman and Gallistel claim that 3- to 5-year-olds honor the how-to-count principles came from their study of how well children adhered to each of the separate principles and how well they coordinated the application of all three principles when responding to requests to count heterogeneous set sizes of 2, 3, 4, 5, 11, and 19 items. The design called for a child to count each set size six times (three times for a linear arrangement and three times for a haphazard arrangement of the display). Thus, it was possible to determine if a child used an idiosyncratic or a standard list. (If they did use an idiosyncratic list, it was not held against them.) Each count trial was scored for whether a child used as many tags as there were items (the one-one principle); for whether the list of tags was systematic, by virtue of the fact that the conventional or an idiosyncratic list was used repeatedly over trials (the stable-order principle); and for whether the child indicated the correct cardinal value of an array by repeating the last tag used. Then, summaries of how well a child did on a given set size were analyzed to determine how many children in each age group honored all or some of the how-to-count principles. Error analyses shed light on the sources of difficulty and development involved in the application of the counting procedure.

The evidence regarding the application of the one-one principle was quite good. A crude index of the tendency to honor this principle is whether or not children used as many tags (unique or not) as there were objects to count. If children attempted to count a given set size, they did quite well at applying either N or ($N \pm 1$) tags even for set sizes of 19. The 3-year-olds did make errors. For set sizes of 7, 9, 11, and 19, 73%, 65%, 67%, and 10% of them used either N or $N \pm$ itags. Still, except for set size 19, these scores are quite creditable.

Error analysis of the application of the one-one principle revealed two major types of errors: (1) the double counting or skipping of an item in the middle of a count; (2) doing the same thing, i.e., double-counting or skipping an item at either the beginning or end of a count. Tagging errors were infrequent, and when they occurred they always involved the repetition of a tag rather than the use of an inappropriate one (e.g., *blue* or a *mouse*). Gelman and Gallistel point out that such results are consistent with a performance-demand hypothesis of the errors. Children have trouble starting and stopping a count (hence, one-too-many or one-too-few tags), and they slip up as they pass between adjacent items, sometimes double counting or skipping an item. They conclude that even 3-year-olds in this experiment did a reasonable job of applying the one-one principle.

More than 90% of the 4- and 5-year-olds and 80% of the 3-year-olds in the Gelman and Gallistel study used the same list on all their trails regardless of set size. Hence, it was concluded that these children honored the stable-order principle. They did not do nearly as well in applying the cardinal principle (see Table 4.1 where the children's tendency to apply all three principles in concert is summarized).

The main reason a child was not scored as having used all three principles was his or her failure to indicate the cardinal value of an array of a given set size. Gelman and Gallistel concluded that the performance demands of counting larger and larger set sizes became too great. Thus, a child forgot to repeat the last tag.

Further Evidence. Some have argued that Gelman and Gallistel granted their subjects too much competence (Siegler, 1979; Sternberg, 1980). But, if preschoolers do "know" the counting principles, they should recognize counting errors. And, if performance demands limit their application of the cardinal principle, experimental manipulations that reduce performance demands should increase their tendency to state the cardinal value of a set. Recent research along both lines lends support to the original Gelman and Gallistel conclusion.

TABLE 4.1
Percentage of Children Who Used All Three How-to-Count Principles in the Gelman and Gallistel Counting Experiment[a]

	Age		
Set Size	*3 Years*	*4 Years*	*5 Years*
2	76	74	93
3	67	79	100
4	57	68	100
5	43	63	86
7	19	47	80
9	0	37	67
11	5	37	47
19	0	16	20

[a] Based on Table 8.10, pp. 128–129, Gelman and Gallistel (1978). These figures included children who applied all principles perfectly and those who were shaky in their adherence to either or both of the one-one and stable order principles. See Chapter 8 of Gelman and Gallistel (1978) for coding information.

Gelman and Gallistel commented on the ubiquitous tendency of their subjects to self-correct their own counts. It is difficult to explain this without presuming that a child monitored the application of the counting principles and detected an error. And, now there is evidence that preschoolers can detect some kinds of counting errors. Fuson and Richards (1979) refer to the fact that 3-year-olds recognize counting errors, although only older children can describe them. Mierkiewicz and Siegler (1981) find that 3-year-olds are able to recognize some counting errors, especially the skipping of an item (what I call a partitioning error). Four- and five-year-olds recognize a diverse set of counting errors (e.g., omitting or adding an extra tag; double counting an item) moreover, they recognize that it is acceptable to count alternate items of the same kind and then back up to count the remaining items of another kind in a given display. They also recognize that it is acceptable to start a count in the middle of an array. Gelman and Meck (in preparation) find that 3½- and 4-year-old children can indicate whether a puppet's count trials had errors when the errors were violations of the cardinal principle (e.g., the puppet said $x + 1$ rather than x in response to a "how many" question). The children did this even for set sizes to which they themselves failed to apply the cardinal principle (e.g., 15 and 20).

Having a puppet do the counting for a preschooler is one way of reducing the performance demands of the task, and this is presumably why the children did so well in detecting cardinal errors. Gelman and Meck (in preparation) followed this reasoning in their second experiment with 2½- to 3-year-olds. As expected, the children's ability to count with accuracy broke down with set sizes larger than 2 and 3. However, when the experimenter did the counting and then asked the child to indicate "how many," 78% of the subjects were correct on at least one set size beyond that which they could count; 44% were correct on set sizes up to 20.

If 3- to 5-year-olds do not have the cardinal principle available, Markman (1979) should not have been able to increase the preschooler's tendency to report the cardinal value of an array as a function of variations in question type. Yet she did. Markman distinguishes between concepts that are organized as *classes* as opposed to *collections* (Markman, 1979; Markman & Siebert, 1976). To illustrate this distinction, consider the concepts of *trees* and *forests*. Given a particular instance of a tree, one can answer whether or not it is a member of the class *trees*. However, given the same instance of the same tree, one cannot answer whether it is a member of a forest. A tree by itself does not a forest make. There must be other trees nearby (i.e., a tree is a member of a forest only if it is in close proximity to many other trees). Likewise, a particular child is not a member of a *family* unless it has a relationship with other people (e.g., siblings or parents). In contrast, a particular child *is* a member of the class *children*.

Markman (1979) suggests that class terms for a given display have the effect of focusing attention on the particular members of the display and that collection terms have the effect of focusing attention on the overall characteristics of the display. She notes that the cardinal number of a display represents the complete set but not the individuals in that set; a set may be said to represent five items but none of the individual items can be labeled *five*.

When young children are asked to count the number of items in a display and then indicate "how many" are there, they have a strong tendency to recount the display (e.g., Schaeffer et al., 1974). Markman tested 3- and 4-year-old children's ability to apply the cardinal principle when asked collection versus class questions. For example, children in the collection condition were instructed: "Here is a nursery school class. Count the children in the class. How many children are in the class?" Children in the class condition were asked: "Here are some nursery school children. Count the children. How many . . . . ?" Set sizes were 4, 5, or 6. On 86% of their trials, children in the collection group gave the last number in their count list as a response to the final question. In contrast, children in the class group were as likely to recount without repeating the last number as they were simply to repeat the last number. Clearly, a standard counting task underestimates the young child's ability to apply the cardinal principle. When all facts are considered, it seems reasonable to say that young children honor the cardinal principle, but this tendency is restricted to certain conditions. And because their application of the cardinal principle depends on applying the one-one and stable-order principles, they obviously must be using these as well—an assumption confirmed by their ability to detect errors in their application.

Implicit Knowledge of the Abstraction and Order-Irrelevance Principles

Over the years, I have varied the type of item used in an experiment, including two- and three-dimensional displays and homogeneous versus heterogeneous displays. I have seen little, if any, effect of these variations on performance levels. For example, Gelman and Tucker report no differences in the ability of their preschool subjects to make absolute judgments of the set sizes of homogeneous of heterogeneous items. Thus, it seems that young children are indifferent to a wider range of variations in item type than predicted by Klahr and Wallace (1976) or observed by Gast (1957). In addition, the Gast study can be faulted on the grounds that children were first tested on homogeneous arrays, which probably prompted the younger children to count only similar items (Gelman & Tucker, 1975).

Elsewhere, I report on experiments designed to determine the conditions under which 3- and 4-year-olds would be affected by item type (Gelman, 1980). In one such experiment, children were asked to count everything in the room. If children refuse to classify animate and inanimate objects together for a count, they should count items within each category only. Given that young children recognize the difference between animate and inanimate objects (Carey, 1978; Gelman & Spelke, 1981; Keil, 1979), it seemed reasonable to expect them to keep these objects in separate groups when counting. The fact that they did not further supports the conclusion that preschoolers apply the abstraction principle when deciding what can be collected together for purposes of a count. In response to instructions, children typically did one of two things. They spontaneously counted all the objects (i.e., people, tables, chairs, etc.) or they started by counting only animate or inanimate objects. But when asked "what about me and you?" (or "what about the other things in the room?"), they continued their count. That is, they did not start over again from 1 as would be expected had they thought that animates and inanimates could not be grouped together for counting.

Given these findings, I conclude that preschoolers are rather indifferent to item types when it comes to applying the counting procedure. I do believe, however, that conditions that make it more difficult for the young child to apply the one-one principle correctly *will* affect performances. I have also reported (Gelman, 1980) on an experiment where children were asked to count the exact same heterogeneous arrays under two conditions. In one condition, they could touch and move the objects; in the other, they could not because the objects were under a plexiglass dome. The idea was that item type would influence performance when items are presented in a way that interferes with the young child's prevalent tendencies to point to, touch, and move objects—a strategy that I see as being used in the service of the partitioning process requirement of the one-one principle. Results were as expected. Overall performance in the plexiglass condition was worse. An age-by-condition-by-set-size interaction supports the hypothesis that practice at counting a given set size reduces the performance demands and hence increases accuracy (cf. Case & Serlin, 1979).

On the basis of these experiments, I conclude that the main effects of stimulus variables involve performance-demand variables or tendencies of young children to be unduly influenced by context variables. Recent habituation studies show that even an infant's ability to discriminate among two-, three-, and four-item arrays is *not* dependent on item type (Starkey, Spelke, & Gelman, 1980; Strauss & Curtis, 1980). Given this fact, it is hardly surprising that the same is true for preschoolers.

In the counting experiment summarized earlier as well as in a subsequent experiment designed to test for the use of the order-irrelevance principle, Gelman and Gallistel asked children to count repeatedly a given set size of heterogeneous items. In both experiments, there was little, if any, tendency to try to keep assigning the same tag to a given item as it got moved around from trial to trial. The children seemed indifferent to the order in which they tagged particular items. Hence, the conclusion followed that children had implicit knowledge of the order-irrelevance principle. And because the subsequent order-irrelevance experiment (see the following section) revealed explicit knowledge of this principle in almost all 5-year-olds and many younger children, the idea that children of the same age have implicit knowledge of the order-irrelevance principle is reasonable.

The Development of Explicit Knowledge

When children as young as 3 are asked to count a set of a given value, over and over again, they are indifferent as to the order of the items as the items change across trials. Such behavior is what one would expect if the child has an implicit understanding of the order-irrelevance principle. The behavior does not index explicit understanding of this principle. Indeed, explicit understanding is at best weak in the 3-year-old child. However, the development of explicit understanding of this principle is well advanced by 5 years of age. This fact is illustrated in the 5-year-olds' performance on a modified counting task.

The modified counting task required a child first to count a linear display of five heterogeneous items. Almost all children do this by starting at one end or another and thereby setting the stage for the modified counting trials. These trials start with the experimenter pointing to some item in the middle of the array and saying: "Count all these but make this be the 1." Having done that, the child is asked to make the designated item the *2, 3, 4* . . . and $x + 1$. Then the child is asked similar questions about a different object. Five-year-olds are nearly perfect across all the modified counting trials. Further, they try to say something about how movement of the items per se does not affect the tagging process.

The results of the modified counting task provides evidence that most 5-year-olds have explicit knowledge of the order-irrelevance principle. The $x + 1$ trial allows us to reach a similar conclusion about the cardinal principle. This is true because many of the 5-year-olds balked at the $x + 1$ request, often complaining "there are only five" and/or "I need another one." Stated differently, they knew their count was conserved no matter how the items were arranged as long as the same set size was maintained through rearrangements of the objects. I submit that they also knew that number names are temporary tags. Otherwise, they would not have rearranged objects in a row so as to establish a correspondence between the position of that item and the order of tags (Merkin & Gelman, 1975). Nor would they have been able to answer the questions asked at the end of the experiment.

To end the modified counting task, the experimenter first pointed out that, over trials, the child had labeled a given object 1, 2, 3, 4, etc. and then another object 1 and 2. The child was then asked if it was all right to use the same count word for the two different objects. Finally, the child was asked if he or she could reverse the names of the object (e.g., by calling the chair a baby and vice versa). Even most of the 5-year-olds failed the Piagetian nominalism question and insisted that a chair was a chair and a baby was a baby. In contrast, the same 5-year-olds—as well as many 3- and 4-year-olds—showed no inclination to restrict the assignment of a given count work. Indeed, they occasionally were very articulate, as was one 4-year-old who said: "It could be 1 or 2 or any number, like 6, 10, and even 14."

Just as there is development from an implicit to an explicit understanding of the order-irrelevance and cardinal count principles, so there is such a course for the other counting principles. As indicated earlier, 3-year-olds can tell which count sequences have double count, omission, and other errors, but only older children can say why (Fuson & Richards, 1979). Mierkiewicz and Siegler (1981) find that preschoolers are able to recognize a variety of counting errors. But it is not until children reach school age that they are able to say why an errorless count sequence involving the alphabet as tags is a better count trial than one that uses the conventional count words but includes errors (Saxe, 1979a). Thus, we see the development of an understanding of the one-one and stable-order counting principles becoming more explicit.

But, it is not only the explicit understanding of the counting procedure that develops. So does an explicit appreciation of the facts that counting is an iterative process and that there is no largest number. Evans (1982) finds that kindergarten children typically resist the idea that each addition of one item will increase number. Their resistance is highly correlated with their ideas of what constitutes a "big number." These are usually under 100 or made-up combinations like "forty-thirty-a hundred." Apparently, children need some experience with largish numbers before they can induce that counting is iterative. For, at a second level of development, children talk about a 1,000,000 and other large numbers when asked what is a very large number. They then allow that the addition of an item can go on and on. But even this advancement does not guarantee that they think there is no upper limit on the natural numbers. Instead, they maintain that despite the possibility of another, and another, and another larger number with each addition of one, there is nevertheless a largest number. Finally, by 8 or 9 years of age, children recognize and accept the possibility of nonending iteration and state that there is no largest number. There seems to be a progressive bootstrapping of one level of understanding to the next with intermediate plateaus where children assimilate enough examples before achieving, in Piagetian terms, a reflective abstraction of their earlier levels of knowledge as well as a new level of understanding.

RELATED NUMBER CONCEPTS AND ABILITIES

There is an evergrowing body of literature on the nature of addition and subtraction skills in preschool children, and it points to the conclusion that preschoolers know that addition increases set size whereas subtraction decreases set size. Smedslund (1966) had 5- and 6-year-olds indicate whether two arrays of equal value ($N = 16$) were in fact equal; then the arrays were screened. When one of the arrays was transformed by adding or subtracting one object, the children were able to indicate which array contained more elements. The same finding was reported for 4- and 5-year-olds by Brush (1972), and for 3-, 4-, and 5-year-olds by Cooper, Starkey, Blevins, Groth, and Leitner (1978). Also, I (Gelman, 1972a, 1972b, 1977) and Cooper et al. (1978) found that 3-, 4-, and 5-year-olds could infer the occurence of a screened addition or subtraction by comparing the pre- and posttransformation values of arrays.

What does one make of the fact that preschoolers can count *and* do understand the respective consequences of adding and subtracting? I submit it is possible for young children to use counting as an algorithm in simple arithmetic tasks. Stated differently, one consequence of being able to count is the ability to develop early skill at addition and subtraction. I often find that 3- and 4-year-olds spontaneously count when confronted with unexpected changes in set sizes and thereby determine the difference. Groen and Resnick (1977) taught 4½-year-olds to solve simple addition problems by use of a counting algorithm. Their instruction consisted of having children first count out two groups of objects of given set sizes, then combine the groups and count them to achieve an answer to arithmetic problems. Half the children spontaneously employed a more efficient algorithm than they had been taught. This was to count on from the cardinal value of the greater of the to-be-added numbers.

Starkey and Gelman (in press) tested 3-, 4-, and 5-year-olds on a variety of mental addition and subtraction tasks. Each task began with the experimenter asking how many pennies she held in her open hand. The experimenter then closed her hand and thereby screened the array of pennies. She then said: "Now I'm putting x pennies in my hand; how many pennies does this bunch have?" or "Now I'm taking x pennies out . . ." Thus, the two values to be added or subtracted were never simultaneously visible. Children did quite well in these tasks. For example, the majority of the 5-year-olds could solve problems that began with one to six items and required adding or subtracting one to four items. As expected, many children used a counting algorithm even though the items were screened.

To be sure, the continued use of a counting algorithm as tasks become more complex could present problems for children in school. The larger a set size, the greater the chance of making counting errors. Written problems are easier to negotiate if some number facts are known or if new algorithms are learned. Obviously, children will have to learn in school many things that they do not know about arithmetic and mathematics. What I am suggesting is that the ability to count facilitates an early understanding of addition and subtraction.

I have hinted at another function served by children's tendency to count. This is that they can provide themselves with thought experiments (cf. Kuhn, 1977) about the nature of natural numbers. Children who set the task of counting all the cracks in the sidewalk, the number of telephone polls they drive by, etc., provide themselves with an opportunity to find out—on their own or from someone else—that counting can go on, and on, and on. This must happen frequently, or else it is hard to explain why half of Evan's first and second grade subjects (who were from a lower-middle to middle-middle class community) said that numbers never end and that there is no largest number. They were not taught about such matters in school.

As indicated before, if children can reach an induction about infinity on the basis of experience with counting, they must have ideas about very large numbers. Counting can serve as a source for learning about the existence of a count sequence that can be very long, as well as for learning about the base rules that contribute to the sequence's potential for length. As in the case of inductions about infinity, I suspect that there is more than one path to this knowledge. Children might ask on their own what the next, and the next, and the next number is. And parents and teachers alike provide input about base rules as to how to count in the 100s and 1000s, etc. Whatever the case may be, the ability and motivation to count at young ages supports inductive learning about some properties of the number system.

In sum, young children who count are able to invent counting algorithms to solve arithmetic problems and provide themselves with practice and inputs that in turn support the acquisition of further knowledge about counting and the natural numbers. Such learning requires a supporting environment. Our culture uses a list of count words and base rules for combining natural numbers just as it provides samples of the English language to the young language learner. However, such learning seems not to require structured lesson plans; it is a case of "informal" learning (cf. Ginsburg, 1977). If I am correct, it should be that counting and simple arithmetic skills are universal and that they develop even in cultures without schools.

EVIDENCE FOR UNIVERSAL ABILITIES

Cross-Cultural Findings

Evidence from a variety of sources converges on the conclusion that the kind of arithmetic abilities we grant preschoolers is universal. First, it appears that most cultures use a counting procedure. It was once commonplace to assign "primitive" numerical abilities to those from nonliterate cultures (e.g., Menninger, 1969). Zaslavsky's (1973) work shows that Africans do indeed count and have done so for centuries. It also illustrates the folly of relying on the ability to use conventional count words as evidence for the ability to count.

Many African societies (e.g., the Kinga, Hebe, and Nyatura of east Africa) use finger gestures and hand configurations to represent different set sizes. A failure to recognize that gestures may be used as tags in enumeration would necessarily lead to an underestimation of the extent to which members of such a society could count. Similarly, a failure to take into account the possibility that number-word sequences need not derive from a base-10 system could lead to the same underestimation of ability. There is a Bushman language that combines the words for 1 and 2 to get the words for 3 and 4. The comparable English count-words sequence for 1 through 4 would read "1, 2, 1-2, 2-2." If we failed to realize that the Bushmen were using a binary concatenation rule, we might conclude they could only count up to 2 and that they had a "one-two-many" conception of differences in set size. Indeed, this is much like the conclusion Menninger (1969) reached. He argued that an ability to count required the use of a count sequence that went beyond the use of 1 and 2: "The number sequence begins at three; three, four, five, . . . etc. When a tribe of South Sea Islanders counts by twos-urapan, okasa, okasa urapan, okasa okasa, okasa okasa urapan (i.e., 1, 2 2'1, 2'2, 2'2'1), we distinctly feel that they have not taken the step from two to three [p. 17]."

Perhaps the best evidence that counting need not be done with a conventional string of words comes from Saxe's (1979a) work in Papua, New Guinea. He reports that people there use the names of their fingers and successive parts of their arms and upper torso as counting tags. The system is illustrated in Fig. 4.1.

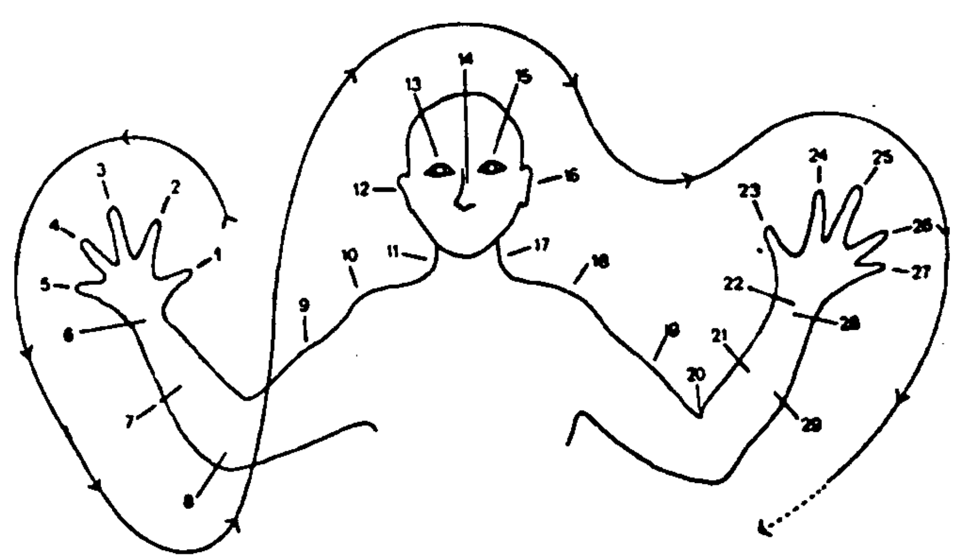

FIG. 4.1. Illustration of the use of body parts to correspond to number words by a group in Papua, New Guinea. Adapted from Saxe (1979a).

Ginsburg and his colleagues lend further support to Zaslavsky's conclusion that Africans do count. They also find that unschooled children in two West African groups—the Dioula and the Baoule—know informal mathematics at about the same level as preschoolers in American culture. For example, they understand the operations of arithmetic and use counting strategies with concrete objects to solve simple arithmetic problems (Posner, 1978). In both communities, children (7–8 years) are able to accomplish such tasks, whether they are in school or not. An effect of a school-nonschooled variable is observed with the Baoule but not the Dioula children. This is because all of the Dioula children are at ceiling on these tasks before they even start school. Posner (1978) attributes this to what she refers to as the informal mathematics in the Dioula culture.

The Dioula are Muslims who have spread throughout the Ivory Coast. Traditionally, they have engaged in commerce and have a well-developed number system (Zaslavsky, 1973). Hence, theirs is a culture wherein informal mathematical notions are indigenous, much like our own. In contrast, the Baoule culture does not emphasize mathematical thinking and thereby provides a less supportive environment for the arithmetic competence of a child to develop. Thus, schooling becomes a significant variable for the Baoule children's performance levels on even simple mathematical tasks.

Ginsburg's (1979) work with inner-city children in the Baltimore and Washington, D.C. areas supports his view that there are "natural" arithmetic abilities that develop without the support of a school environment. When tested with tasks that assessed their understanding of *more*, and their ability to count and to add using a counting algorithm, these children showed the same kinds of errors as did middle-class children here and African children in West Africa. The implication here is that a common error pattern reflects a common underlying capacity.

Saxe's (1980) work with the Oksapmia of New Guinea provides another example of how the presence of a supporting environment enhances the level of arithmetic ability in an culture without schools. Until recently, the Oksapmia had no money. Now some men are flown to work on a tea plantation and return home with money. Some have even opened small trading stores. Preliminary results show that those who have had the greatest interaction with currency have developed, on their own, more efficient calculation algorithms than those who have not had the interaction. Furthermore, the most skilled individuals are beginning to introduce a base system that is not present in the original count system shown in Fig. 4.1.

I must emphasize that neither I nor Ginsburg are claiming that schooling has no effect on the development of mathematical abilities. Our view is that children bring a great deal of knowledge about numbers and arithmetic to the school setting because counting and simple notions of addition, subtraction, equivalence, and nonequivalence reflect natural, universal abilities. These develop in a supporting environment.

The Effect of Retardation

From the evidence in the previous section, one might conclude that there are limited individual differences in the abilities to learn to count and use counting to solve simple arithmetic tasks. Similar lines of evidence are often cited to support the conclusion that there are limited effects of individual differences in the ability to acquire a first language. In the case of language acquisition there are, however, effects of an extreme variation in individual differences. Retarded individuals are often delayed in the start of language acquisition (Lenneberg, 1966) and, in some cases, lag far behind their Mental Age (M.A.) controls. Fowler, Gelman, and Gleitman (1980) find very limited syntactic abilities in some Down's syndrome teen-agers. Still, many of their abilities are indistinguishable from those of a control group. For example, when teen-age retardates with an average mean length of utterance (MLU) of about three words are compared to normal 2½- to 3-year-olds with the same MLU, we find no differences in the kinds of grammatical morphemes and syntactic structures used.

Reasoning by analogy from the language-acquisition data, I thought it possible that retarded children would show a comparable delay in the acquisition of their ability to count and hence to solve the kind of simple arithmetic tasks considered in this chapter. To find out, Gelman, Haberstedtt, and Hungerford (in preparation) assessed the counting ability of a sample of retarded children. Then,

Starkey and Gelman (in preparation) assessed the ability of some of these children to solve the same arithmetic problems that we had given to normal preschoolers. Children came from three class rooms at a parochial school. The median ages of the groups were 7, 11, and 13 years, respectively, and their median M.A.s were 4 years, 3.5 months; 5 years 9 months; and 7 years 6 months. Half of the two younger groups and one individual from the oldest group were Down's syndrome children. All children in each group were seen in the counting study, and then a sample of the older children was selected for the arithmetic study.

One condition in the counting study was run in much the same way as the initial Gelman and Gallistel experiment—a second demonstration condition was run after the basic condition. In pilot work, I noted a limited tendency for the retarded children to point at and move objects. Hence, we added the *demonstration* condition, wherein the experimenter asked the children to count as she did (i.e., "to touch each toy and count out loud—just like this"). Both sessions were recorded for later scoring according to the Gelman and Gallistel code.

In Fig. 4.2, the results of a composite counting analysis are shown as a function of set size, M.A. group, condition, and criterion strength. The left-hand panels show the percentage of children who used all three how-to-count principles on at least one of the three test trials; the right-hand panels show the percentage who counted perfectly. Inasmuch as no child used systematic nonconventional lists, these percentages reflect tendencies to count correctly with the standard count sequence.

There is a clear effect of M.A. Children in the youngest group with an M.A. of 4 years 3 months are not able—under any conditions—to count even set sizes of 2 and 3 accurately. When left on their own (i.e., the no-demonstration condition), around 30%, 20%, and 0% of these children consistently count set sizes of 2, 3 and 5. The comparable figures reported by Gelman and Gallistel for normal preschoolers were better. For 3-year-olds, they were 76%, 67%, and 58%; for 4-year-olds, they were 74%, 79%, and 68%. Furthermore, some normal 3-year-olds could count set sizes of 7, and some 4-year-olds could succeed on set sizes of 7 through 19. In terms of developmental level, the retardates were behind what I expect of normal 3- and 4-year-olds. The tendency toward a larger developmental lag than expected according to M.A. level persists throughout the groups.

The group data represent individual children who seem unable to count at all and children who are quite excellent counters. Principle-by-principle analyses of the data revealed that retarded children who failed our counting criteria produced error types that I have not see in normal preschoolers. We failed to observe the use of *any* idiosyncratic lists, either within repeated trials on the same set size or across trials. Further, we observed the repeated use of a given count word (a tagging error in the use of the one-one principle), some labeling of objects by name (another tagging error), and a ubiquitous tendency to tag items repeatedly

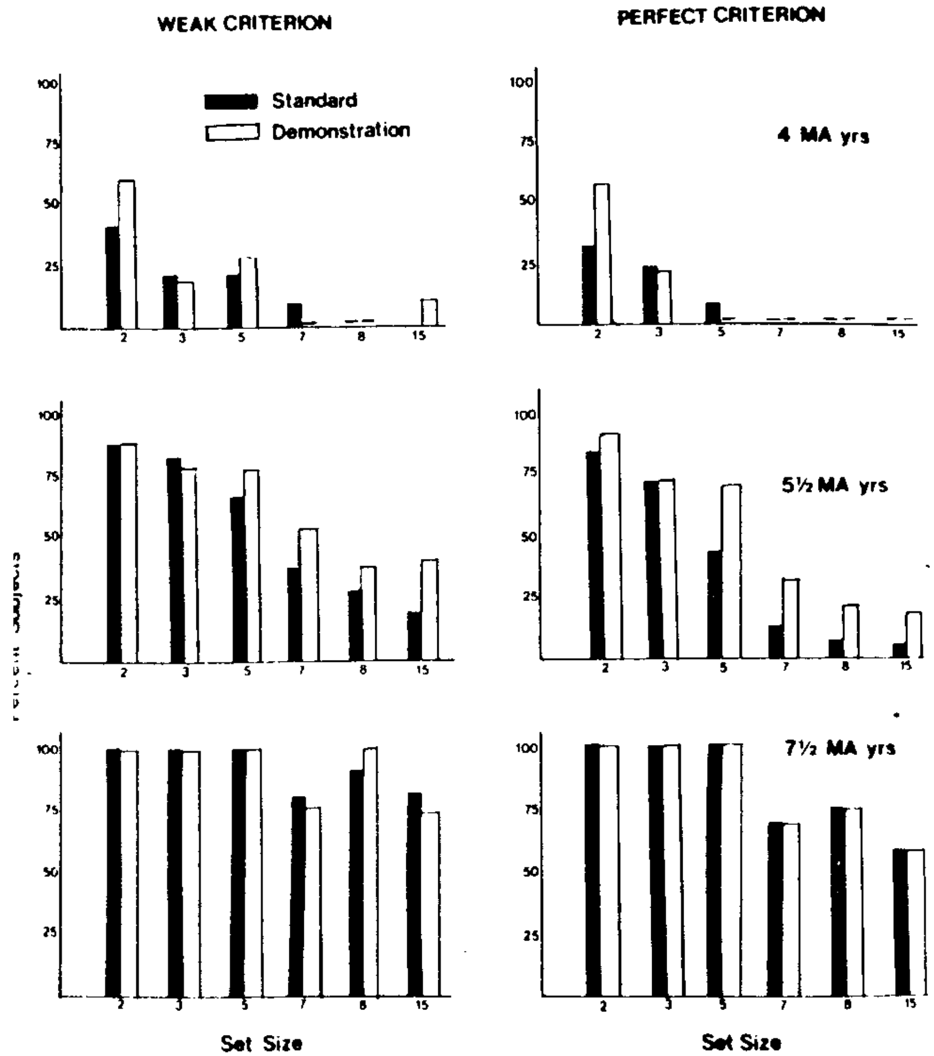

FIG. 4.2. The ability of retarded children to apply all three how-to-count principles as a function of M.A. level, set size, and experimental condition.

that had already been tagged. When asked "how many" items there were in an array, children had a strong tendency to keep saying the same number (usually 1 or 2) across different set sizes. Occasionally, a child even used the name or an attribute of an object.

In order to determine whether the ability to count is related to the ability to solve simple arithmetic problems, we assigned an overall grade of "poor," "shaky," or "very good" to each child. Eighteen children, six within each grade, were then seen in conditions much like those used by Starkey and Gelman (in press) with normal preschoolers. As we wanted to use some additional tasks, we also ran groups of normal 3-, 4-, and 5-year-olds for comparison with the retardates.

The poor counters (median M.A. = 6 yrs.; median C.A. = 12 yrs.) in the arithmetic study simply could not count set sizes larger than 5. And only one of them was able to indicate the cardinal number for all three of the smaller set sizes (2, 3, 5). Furthermore, their lists of count words were often random beyond 5, similar to what Fuson and Richards (1979) call "spews." Shaky counters (median M.A. = 6 yrs. 6 mos.; median C.A. = 13 yrs.) used the conventional sequence most of the time. However, as set sizes increased beyond 5, they often used the wrong number of tags and hence were scored as weak in their application of the one-one principle; they typically failed on the cardinal principle. Very good counters (median M.A. = 8 yrs. 3 mos.; median C.A. = 13 yrs. 3 mos.) were able to use all three how-to-count principles for all set sizes. Except for one or two error trials on the larger set sizes, these principles were applied consistently. When I watch these children count, the qualitative impression is that the poor group counts by rote, the shaky group is catching on, and the good group is just that. Performance differences on simple arithmetic tasks confirm these impressions.

Here I summarize the data for only the simple arithmetic tasks and some repeated iteration tasks. These involved initial arrays of one to six pennies, which were transformed by adding or subtracting one to five pennies. The initial arrays were screened before addition or subtraction took place.

For both normal preschoolers and retarded children, the difficulty of the simple addition and subtraction problems increased as a function of set size, and success was related to counting ability. Actually, the retarded children did somewhat better than the normal younger children. Thus, 3-year-olds were correct on 31% and 35% of their addition and subtraction tasks, and the respective pairs of percentage correct scores for the 4- and 5-year-olds were 64% versus 52% and 83% versus 71%. In contrast, the poor retarded counters were correct on 64% and 40% of the subtraction problems, shaky counters on 72% and 49%, and good counters on 97% and 97%.

I suspect that many of the differences between the results for normal 3-year-olds and the poor counters reflected the use of different solutions. The 3-year-olds counted and did about as well as expected, given that their skill at counting breaks down around 3. The poor counters in the retardate sample, unlike the normal 3-year-olds, had the benefit of drill in school on similar arithmetic tasks and had memorized some number facts. The suggestion that different solution types were used is confirmed by error analyses.

No matter what their answers, normal preschoolers at every level gave a larger number than the augend on addition trials and a smaller number than the minuend on subtraction trials. This was not true for the retarded poor counters who often gave addition answers for subtraction problems. As an example, they might say the answer to a $4 - 2$ problem is 6. Furthermore, iteration addition tasks (e.g., $x + 1 + 1$) were easier than control items (e.g., $x + 2$) for all preschoolers. This would be expected if solutions were reached via a counting algorithm and not by the retrieval of memorized number facts. Inasmuch as there was such an advantage on the iteration task for the shaky counters but not for the poor counters, the influence that the poor counters did not use a counting strategy is supported.

In sum, as expected, we are finding differences in the ability to count as a function of M.A. in a retarded population. And the hypothesis that these in turn reflect differences in the understanding and solution types used on simple mental arithmetic tasks is supported. On the basis of some pilot work, I venture to guess that the retardate's problems with money are, (e.g., they have terrible problems shopping or buying tickets,) likewise related to a failure to understand the counting principles. Indeed, Thurlow and Turnure (1977) suggest that special education programs probably fail in teaching money concepts because the programs assume the ability to count. In the context of the present chapter, I find Thurlow and Turnure's (1977) speculation that "most [normal] children are apparently able to pick up much of their knowledge about time and money from casual or incidental exposure to the concepts [p. 203]" intriguing because it suggests that these abilities are also candidates for natural and universal abilities. At least with regard to money, Saxe's and Zaslavsky's work would support this conjecture.

Counting in Babies? Probably

Studies of infant's abilities to abstract the numerical value of arrays lend further support to the view that number is a natural domain of competence. Starkey and Cooper (1980) showed infants aged 4 through 6 months linear arrays of dots of white light. In the first phase of their experiment, infants were repeatedly shown a given set size in arrays that varied in length and density over habituation trials. Infants dishabituated to changes in numerosity from two to three or three to two but not changes in length on density. In a subsequent study, Starkey and Cooper (1980) found that 6- through 8-month-old infants habituated to displays of three (or four) items and dishabituated when shown displays of four (or three) dots.

Recovery of habituation is often taken as an index of infants' ability to discriminate between the display they habituate to and the subsequent display. A follow-up to the Starkey and Cooper work confirms the assumption made by these investigators that the reported discriminations were based on numerical judgments. Starkey, Gelman, and Spelke (1980) tested 6- to 8-month-old babies with heterogeneous displays. The displays were photographs of common household items (e.g., comb, pipe, lemon, scissors, corkscrew, etc.) selected to include a variety of colors, shapes, sizes, and surface textures. Each array contained either two or three objects, and no two objects in any array were the same.

Further, the spatial arrangement of the objects was unique from trial to trial. In short, the only common characteristic of the set of three-item and two-item displays was the numerical value. Half of the babies in the experiment viewed either a set of two-item or three-item displays. When their tendency to look at these arrays habituated, they were given a set of posthabituation trials. Both the two-item and three-item groups were shown two-object and three-object arrays presented in alternation. We predicted that infants who habituated to two-item arrays during phase one of the experiment would look longer at the three-item array during posthabituation trials. Conversely, infants habituated to three-item arrays should look longer at the two-item arrays.

As predicted, during the recovery phase, infants looked longer at *their* different-number arrays than at their same-number arrays. Strauss and Curtis (1980) have reported a similar result. In addition, they found that female infants habituate to the class of three (or four) objects and then recover to a change to four (or three) objects. The conclusion I reach about such findings is that infants can attend to the number of objects in a display and abstract a numerical invariant over changes in displays. Inasmuch as we intentionally varied item types and item positions, it is hard to imagine what else they could have been responding to.

In further studies, Starkey, Spelke, and I have determined that infants can also to respond intermodally to numerical information. One study used a procedure devised by Spelke (1976) to investigate infants' knowledge of an auditory-visual relationship. An infant is shown two films side by side while, between them, a loudspeaker plays the sound track that goes with one of the movies. Because infants look at the appropriate movie (i.e., the one that corresponds to the sound track), Spelke has been able to investigate the nature and development of intermodal perception in infants (e.g., Spelke, 1979). In our study, infants were shown two-item and three-item heterogeneous displays placed side by side. The loudspeaker between the displays emitted either two or three taps on each trial. I confess being surprised to find that babies had a significant tendency to look at the two-item display when two taps were sounded and at the three-item display when three taps were sounded. Thus, it is not only that babies attend to the numerical value represented in a visual display; they can also match visual and auditory modes of presentation on the basis of number. To do this, they must not only be able to abstract number but also be able to use a rudimentary form of nonverbal counting. For what other procedure can be used to compare visual and auditory presentations?

I said I was surprised by the intermodal results. To be sure, they lend strong support to the thesis I advance here. But even if I wanted to say that the full-blown ability to count is innate (and I do not), I need not expect infants to attend to and use number. The human ability to walk upright is largely innate; yet, no one expects a 6-month-old to walk. Hence, I'm puzzled by the fact that 6-month-old infants are interested enough in number to succeed on our tasks. In any event, the research with infants lends support to the idea that the ability to abstract numerical values of displays, and do so by something akin to counting, is natural.

FINAL COMMENTARY

I have argued that the abilities to count and to do simple arithmetic tasks are natural, universal abilities. The preschooler's acquisition of counting is guided by a set of counting principles. Babies can match a numerical abstraction of small sets in the visual mode with one in the auditory mode. As best as we can tell, normal people in all cultures are able to count. And even in environments without schools, there is evidence that the people can solve simple arithmetic tasks. Finally, the ability to count is diagnostic with regard to retarded children's ability to solve simple mental arithmetic problems.

I have repeatedly used the phrase "simple arithmetic" tasks and have done so on purpose. I do not want people to reach the conclusion that rich mathematical abilities can develop irrespective of the environment. Even if I am correct in assuming that there are universal arithmetic abilities, it does not follow that the teaching of mathematics is unimportant. To illustrate why, I consider another natural universal ability—the understanding and production of speech.

Every normal child can and does acquire his or her mother tongue. Language learning, although dependent on a supporting environment, seems to be able to proceed without structured lesson plans (e.g., Newport, Gleitman, & Gleitman, 1977). Still, grammar is taught in schools. This, I submit, is because educators recognize a basic distinction between the ability to converse and the ability to access the structure of the spoken language. Grammar lessons are geared to teaching children the rules that govern their language. However, mastery of these rules does not guarantee that one can do linguistics. This is the task of professional linguists who have studied long and hard to achieve their special abilities. Similar considerations apply to the acquisition of mathematical prowess. The child who invents a counting algorithm is unlikely to discover, on his or her own, the formal properties of a group. Like linguists, mathematicians need to study and master a great deal of mathematics before they are able to do mathematics. And as compared with the ability to learn to count, there are tremendous individual differences in mathematical ability. It remains an open question as to how and whether early arithmetic abilities are related to the ability to learn mathematics in school.

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