CAN A DECAY PROCESS EXPLAIN THE TIMING OF CONDITIONED RESPONSES?

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To explain time-scale invariant distributions of response latencies, it appears to be necessary to postulate scalar noise in the remembered intervals, against which the subjective measure of the currently elapsing interval is compared. At least in some cases, the observed variability cannot be due to variability in the subjective intervals written to memory; it must come from noise (variability) in the reading of a memory. The Staddon and Higa proposal offers no explanation for the observed variability, and it is unclear what noise assumption would yield the observed variability, given their assumption that intervals are timed by a nonlinear decay process. The decay process cannot plausibly be represented by the logarithmic function, because it begins and ends at infinity. The assumption of any form of nonlinear timing is inconsistent with the most important result of the time-left experiment, which is that the changeover time increases linearly with the comparison-standard difference.

Key words: subjective time, time-scale invariance, scalar variability, response latencies, temporal psychophysics

The data that Gibbon's scalar expectancy theory (SET) tries to explain are primarily data on the latencies of conditioned responses. Conditioned responding tends to begin when a certain proportion of the reinforcement latency has elapsed. My comments focus on the assumptions crucial to the explanation of these response latencies.

The Origins of the Noise in Temporal Decision Making

The opening and closing parts of Staddon and Higa's article seem to imply that the principal problem with Gibbon's SET is that it does not enable us to derive the variability observed in the latencies of conditioned behavior. To explain these response latencies, SET assumes a timer, a memory that stores outputs from the timer, and a comparison process. The comparison process generates a response when the ratio of the current value from the timer to the comparison value retrieved from memory exceeds a threshold. To model the timer, Gibbon suggested a Poisson pacemaker feeding an accumulator. The essential feature of this timer is that subjective time (the internal measure of the duration of an interval) is proportional to objective time.

Staddon and Higa suggest that the theoretically relevant property of this model for the timer is what it predicts about the variability in the repeated timing of the same interval. They note that the *variance* in subjective time for a given duration of the interval being timed will be equal to the mean value of the obtained accumulations. If the variance is proportional to the mean, then the *standard deviation* is proportional to the square root of the mean. Thus, the greater the mean, the smaller the standard deviation in proportion to the mean. In his original formulation of SET, Gibbon (1977) made this same point, and he drew the necessary conclusion, namely, that Poisson variability in accumulations (subjective intervals) could not explain the scalar variability in the response latencies. That is, it could not explain why the standard deviations of the obtained distributions are proportional to the means (and, more generally, why normalized distributions, regardless of their shape, are superimposable). It is unclear why this issue is revisited at this late date; it has been a settled issue from the beginning.

Gibbon (1977) suggested reasons why proportional variability might occur, focusing on possible trial-to-trial variations in the rate at which the pacemaker ran. Subsequent work, however, has shown that the scalar variability in

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the response distributions cannot be due primarily to variability in the original timing of the remembered intervals. Rather, it must be due to variability in the process of reading a value stored in memory in order to use it as the comparison value in a decision. The necessity of attributing the variability in response latencies primarily to the memory reading process is shown most clearly by the results obtained from the two-standard version of the time-left task, because in this experiment, the central tendency or expectation read from memory has no counterpart in the input intervals. None of the input intervals is anywhere near the expectation of the distribution of those intervals. Thus, the scalar variability must arise from the process of reading the expectation itself, not from the distribution of intervals on which that expectation is based.

In the time-left task, the subject compares the time left until reward on one side (the so-called comparison or time-left side) against the unvarying expectation on the so-called standard side. The delay on the time-left side gets shorter as the trial proceeds, while the delay on the standard side does not. When the subject estimates that the time left on the comparison side is shorter than the delay on the standard side, it switches from the standard side to the comparison side. In the two-standard version of this task, the expectation on the standard side comes from an experienced population of two randomly intermixed standard delays: On some trials, the standard delay is very short, for example, 15 s; on others, it is very long, for example, 240 s. The subject never knows which delay is in force on the standard side on any trial, so it must base its behavior on the expectation it computes from the bimodal population of two very different delays. The time left at which the subject changes over tells us what the subject takes to be the expectation or central tendency in this bimodal distribution of delays on the standard side. The changeover turns out to occur at about the harmonic mean (e.g., Brunner, Gibbon, & Fairhurst, 1994), which, for the above illustrative values of the two standard intervals, is 30 s.

The crucial result for present purposes is that the distribution of changeover responses (the distribution of elapsed times at which the subject switches from the standard side to the comparison side) is essentially the same in the two-standard case as it is in the case in which there is but a single standard interval at 30 s. Thus, the distribution of changeover responses cannot be a reflection of variability in the timing of the very short and very long intervals that compose the population of standard intervals. The population in the two-standard version is very different from the population in the one-standard version. Only the expectations are the same in these two cases, not the populations on which those expectations are based. Yet, these expectations produce the same distribution of changeover latencies. One is forced to conclude that the proportional standard deviation (the scalar variability) observed in the distribution of response latencies reflects trial-to-trial variability in the target values retrieved from memory rather than trial-to-trial variability in the original inputs to memory (the "accumulations" in the accumulator).

Put another way, one is forced to conclude that time-scale invariance is a basic property of the noise in a remembered temporal interval, no matter how that interval was originally derived, whether directly from the timer or from a computation of the expectation of a distribution. Time-scale invariance means that one cannot deduce the time scale of the experiment from the distribution of response latencies. That is, from looking at this distribution, one cannot estimate what the objective duration of the interval to be remembered was. If the variability in the observed distribution were not proportional to its mean, then one would be able to deduce the time scale of the experiment (the reinforcement latency) in the absence of a scale factor for the x axis (numerically labeled ticks). The proportion between the width of the distribution and its location along the x axis would indicate the time scale of the experiment (the reinforcement latency). In short, the assumption of Gaussian noise in the signal that comes from memory, with a standard deviation proportional to its mean value, is indeed a postulate in SET, not a deduction. It is best seen, however, as a manifestation of a more general property of conditioned behavior, namely, time-scale invariance.

The postulation of scale-invariant noise in the values read from memory is a "deeper problem" with SET (Staddon and Higa, p. 227) only if there are other assumptions about the sources of variability, from which the variability in response latencies may be deduced. No such assumptions are presented in this paper. In fact, Staddon and Higa refuse to say anything about the characteristics of the noise in the signals that determine behavior in their model. A problem I have with any proposal that the subjective measure of time is given by a decay function is that this makes it difficult to obtain a time-scale invariant model. The assumed rate of decay imposes a time scale, making it difficult to have a time-scale invariant model.

One could explain the scalar variability of response latencies if one postulated (a) that subjective time is a logarithmic function of objective time and (b) that the variability or noise in the subjective time signal is independent of the magnitude of that signal (i.e., constant, rather than varying as a function of signal strength). However, as explained below, the postulation of a logarithmic relation is incompatible with the assumption that time is measured by a decay process, which appears to be the foundational assumption in the Staddon and Higa model. Second, the assumption that the noise in the signal is independent of signal level is physically implausible. Finally, this physically implausible postulate is just as much a postulate as the postulation of scalar variability in SET. Thus, the Staddon and Higa model does not derive the observed scalar variability from more basic assumptions. Indeed, it does not even provide an explanation of the observed scalar variability.

The Function Relating Subjective Intervals to Objective Intervals

The essential feature of the accumulator model of the timer is that the subjective interval (the quantity in the accumulator at the end of an objective interval) is proportional to the objective interval. SET would not be in any consequential way altered if the Poisson pacemaker assumption were abandoned, leaving only the foundational assumption that subjective intervals are proportional to objective intervals. Empirically, this function appears to be a linear function rather than a strictly proportional one, but the deviation of the intercept from the origin is small, so, for most purposes, the function can be treated as one of simple proportionality, which is what I will assume hereafter.

Staddon and Higa suggest an alternative to this assumption. Actually, they suggest at least two alternatives, possibly three. One alternative is that the subjective interval decreases in accord with the sum of several exponentially decaying terms (their multiple-time-scale function).[1] This seems to be the assumption that they are in the end most deeply committed to. The other alternative, which they suggest is equivalent to the first for practical purposes, is that the subjective quantity (the quantity in the head) that corresponds to an objective interval decreases as the negative logarithm of the objective interval. A third suggestion, entertained at various points in their argument, is that the relation is a power function with a negative exponent. I do not agree that these alternatives are for practical purposes equivalent. This makes it hard to assess the viability of Staddon and Higa's proposal, because they sometimes rest their argument on properties of the logarithmic function, while at other times they assume that the sum-of-exponentials function best describes the relation. These are mutually incompatible assumptions. The claim (on p. 220) that "There are several other functions that have very similar properties to the logarithmic: power . . ., the sum of exponentials, and others . . . (Figure 1)" is not defensible, either on the grounds by which the properties of functions are usually compared or on the grounds that the differences between the functions are not great enough within a reasonable range of intervals to matter in practice.

One important property of a function is its behavior at the extremes of its argument. The negative logarithmic function of time [$\tau = -\log(t)$] goes to plus infinity as time goes to zero. This property, all by itself, is an obstacle to the assumption that this function could describe the quantitative relation between objective time intervals and the signals (or traces) in the head that they give rise to. Moreover, as time goes to infinity, the nega-

[1] It may seem odd to speak of a decreasing measure of an increasing function, but that is inescapable in a model that uses the state of decay to measure the magnitude of an increasing variable (as in, e.g., carbon 14 dating). As Staddon and Higa note, this inverse measure (a measure that gets smaller as the thing being measured gets bigger) is more or less okay, as long as the relation between the quantity measured and the measure is monotonic. However, one consequence of using a decay measure is that the measure must be nonlinearly related to the thing measured. This leads to problems, as explained below.

tive log function goes to minus infinity. This is a decidedly odd property for a "decay" process to have. Usually, the more something has decayed, the closer its absolute value is to zero. For time intervals greater than one, the more the negative logarithm "decays," the farther its absolute value gets from zero.

The power function also goes to plus infinity as time goes to zero (again an obstacle to postulating it), but at least it goes to zero as time goes to infinity, as a decay function should. This means, however, that as time intervals get larger, the difference between any logarithmic function of time and any power function of time becomes arbitrarily large. Thus, these two functions are not interchangeable over any very large range of intervals. (Remember that a straight line is a good approximation to any smooth function over a short enough interval.)

Finally, Staddon and Higa's MTS function—a sum of decaying exponentials—differs by arbitrarily large amounts from the negative log function at both ends. Any sum of exponentials is finite at both extremes of its argument, whereas the logarithmic function is infinite at both extremes.

Another very important property of the proposed functions in the present context is how they relate objective differences and ratios to subjective differences and ratios. SET proposes that subjective time is proportional to objective time. More formally, it is a scalar function of objective time, that is, $\tau = kt$, where τ represents the subjective duration of an interval, t is its objective duration, and k is a constant of proportionality (scaling factor). This relation has the unique and theoretically very important property that equal objective differences map to equal subjective differences and equal objective ratios map to equal subjective ratios. This means, for example, that the difference between the subjective durations corresponding to objective durations of 10 and 20 s is the same as the difference between the subjective durations corresponding to objective durations of 40 and 50 s. Thus, equal differences in the world map to equal differences in the head. And similarly, the ratio between the subjective durations corresponding to objective durations of 1 and 10 s is the same as the ratio between subjective durations corresponding to objective durations of 5 and 50 s. Thus, equal ratios in the world map to equal ratios in the head. (Formally, this is because, if $t_1 - t_2 = t_3 - t_4$, then $kt_1 - kt_2 = kt_3 - kt_4$, and if $t_1/t_2 = t_3/t_4$, then $kt_1/kt_2 = kt_3/kt_4$, where $t_1 \neq t_2$ and $t_1 \neq t_3$.) All of the other proposed functions (mappings from the world to the head) lack one or both of these two important properties.

The power function does not carry equal objective differences into equal subjective differences. Assuming again that $t_1 \neq t_2$ and $t_1 \neq t_3$: If $t_1 - t_2 = t_3 - t_4$, then $t_1^p - t_2^p \neq t_3^p - t_4^p$. For example, $1^2 - 2^2 \neq 2^2 - 3^2$. However, the power function does carry equal objective ratios into equal subjective ratios: If $t_1/t_2 = t_3/t_4$, then $t_1^p/t_2^p = t_3^p/t_4^p$. For numerical illustration, note that $1^2/2^2 = 2^2/4^2$. Neither of the other two functions has this property.

The log function does not carry equal objective differences into equal subjective differences or equal objective ratios into equal subjective ratios; rather, it carries equal objective ratios into equal subjective differences (a property that Staddon and Higa make extensive use of). Thus, under the same conditions on the ts as above, if $t_1 - t_2 = t_3 - t_4$, then $\log(t_1) - \log(t_2) \neq \log(t_3) - \log(t_4)$, and if $t_1/t_2 = t_3/t_4$, then $\log(t_1)/\log(t_2) \neq \log(t_3)/\log(t_4)$. However, when $t_1/t_2 = t_3/t_4$, then $\log(t_1) - \log(t_2) = \log(t_3) - \log(t_4)$. For numerical illustration, note that $\log(1) - \log(0) = 0 + \infty \neq \log(2) - \log(1) = 0.3$, and $\log(2)/\log(1) = 0.3/0 = \infty \neq \log(4)/\log(2) = 0.6/0.3 = 2$. However, $\log(2) - \log(1) = 0.3 - 0 = \log(4) - \log(2) = 0.6 - 0.3$.

The exponential function has the inverse property; it carries equal objective differences into equal subjective ratios. Under the same conditions on the ts as above, if $t_1 - t_2 = t_3 - t_4$, then $b^{\alpha t_1} - b^{\alpha t_2} \neq b^{\alpha t_3} - b^{\alpha t_4}$, and if $t^1/t^2 = t^3/t^4$, then $b^{\alpha t_1}/b^{\alpha t_2} \neq b^{\alpha t_3}/b^{\alpha t_4}$. However, if $t^1 - t^2 = t^3 - t^4$, then $b^{\alpha t_1}/b^{\alpha t_2} = b^{\alpha t_3}/b^{\alpha t_4}$. For numerical illustration, note that $2^2 - 2^1 \neq 2^3 - 2^2$ and $2^2/2^1 = 2 \neq 2^4/2^2 = 4$. However, $2^2/2^1 = 2^3/2^2 = 2$.

A sum of exponentials is not itself an exponential function, so the MTS function suggested by Staddon and Higa does not possess any of these potentially useful properties. The function that Staddon and Higa graph as the MTS function in their Figure 1 was obtained by a simulation in which the weighting of the different exponential terms was free to vary. The actual weighting that produced the graph is apparently not known. However, the following power, logarithmic, and sum-of-exponentials functions produce curves as close or closer to each other than those in their Figure 1. The power function is the one Staddon and Higa assumed. The exponentials have the decay constants that they assumed. The negative log function has been scaled and displaced vertically so as to be as close to these two functions as possible over the range covered by Staddon and Higa's Figure 1. Thus, these functions allow us to estimate the extent to which the MTS function has the properties of the logarithmic function within the range graphed by Staddon and Higa (the situation is much worse outside this range): power: $\tau = t^{-0.45}$; negative log: $\tau = -0.39\log(t) + 0.8$; sum of exponentials: $\tau = \mathrm{MTS}(t) = (1/2.5)(e^{-0.36t} + e^{-0.105t} + 0.5e^{-0.006t})$. From the third function, we can calculate that $\mathrm{MTS}(1) - \mathrm{MTS}(2) = 0.12 \neq \mathrm{MTS}(3) - \mathrm{MTS}(4) = 0.07$ and $\mathrm{MTS}(1)/\mathrm{MTS}(10) = 2.47 \neq \mathrm{MTS}(5)/\mathrm{MTS}(50) = 3.302$, and moreover, $\mathrm{MTS}(1)/\mathrm{MTS}(31) = 4.62 \neq \mathrm{MTS}(20)/\mathrm{MTS}(50) = 1.508$ and $\mathrm{MTS}(1) - \mathrm{MTS}(10) = 0.5 \neq \mathrm{MTS}(5) - \mathrm{MTS}(50) = 0.35$. These numerical examples show, when time is measured by a sum of decaying exponentials, that equal objective differences do not correspond to equal subjective differences, equal objective ratios do not correspond to equal subjective ratios, equal objective differences do not map to equal subjective ratios, and, finally, equal objective ratios do not map to equal subjective differences (as they would if the MTS function could be substituted for the log function and vice versa).

The numerical examples given above limit the values of t to the range graphed by Staddon and Higa in their Figure 1, and yet the discrepancies between the functions are substantial—great enough to yield measurably different predictions even when the range of t is thus limited. In fact, however, SET applies to experiments covering a considerably broader range—from a few seconds to 3,000 s (about three orders of magnitude). Over this range, the differences between the functions that Staddon and Higa suggest are equivalent for practical purposes are very large and completely unmistakable. For example, when $t = 1{,}000$, the MTS function in Staddon and Higa's Figure 1 is already effectively zero, whereas the power function is 0.04 and declining very slowly, and the vertically displaced negative logarithmic function is well below zero at -0.37 and growing ever more negative (ever farther from the zero value at which a decay function ought to terminate). Thus, before we can evaluate Staddon and Higa's proposals, they will have to settle on a form for the relation between subjective intervals (the signals in the head) and the objective intervals, because the predictions of an MTS model cannot in fact be reasonably approximated by the predictions of a logarithmic model and vice versa.

The relation between differences and ratios in the subjective realm and differences and ratios in the corresponding objective quantities is fundamental to the issues raised by Staddon and Higa's article. We are here concerned with contrasting models for the processes in the brain that "process," "operate on," or "do computations with" subjective intervals to produce behavior. The subjective intervals are signals or traces in the brain that are at least monotonically related to objective intervals, and therefore can "encode" those intervals. Models for the processes in the brain that determine the timing of conditioned behavior are evaluated on the basis of how well they predict the timing of the animal's responses given various objective intervals (usually reinforcement latencies). The predictions depend jointly on the postulated quantitative relation between the subjective intervals and the objective intervals, on the form and sources for the noise in these neural signals, and on the formal properties of the operations or processes in the brain into which these subjective intervals enter in order to determine the observed behavior.

Many experimental results in the timing literature are very accurately accounted for by a model that makes the defining assumptions in SET, which are that (a) subjective intervals are proportional to objective intervals; (b) the noise in remembered intervals is Gaussian with a standard deviation proportional to the interval being remembered; and (c) the decision variable—the quantity that generates a response when it exceeds a decision threshold—is a ratio of subjective intervals. Put another way, this last assumption is that the measure of the similarity of two intervals is the ratio of their subjective measures. A ratio of one indicates perfect similarity. Decisions to respond occur when this measure of similarity exceeds a threshold, which is usually appreciably less than one. Decisions to stop responding occur when this measure of similarity is appreciably greater than one. The success of SET at giving quantitatively accurate explanations of the experimental data is evidence for the correctness of its assumptions, including, of course, the assumption that subjective intervals are proportional to objective intervals. However, the experimental evidence most directly relevant to this particular assumption comes from the time-left experiment, to which Staddon and Higa's discussion does not do justice.

The fundamental idea behind the time-left experiment is that if subjective intervals are proportional to the logarithms of objective intervals, then when the brain subtracts one such quantity from another, it is equivalent to dividing the corresponding objective intervals. The subjective result of this operation (the signal generated when the signal for the elapsed interval is subtracted from the signal for the comparison interval) corresponds to the dimensionless quantity that is obtained by dividing one objective interval by another, that is, to the ratio of two objective intervals. This follows directly from the property of the logarithmic function stressed above, namely that it carries equal objective ratios into equal subjective differences. Thus, differences in the head correspond to ratios in the world.

Because a logarithmic encoding of objective intervals converts subjective subtraction (subtraction in the head) into objective division, the assumption of such an encoding makes startling predictions about what will happen when a subject is faced with a task in which it must compare a subjective interval obtained by subtraction with another, separately specified, subjective interval. This is what the time-left task does. The subject must compare the time left until reward is obtained on the so-called comparison (C) side with the standard (S) delay of reward on the other side. The time-left to reward on the comparison side gets shorter as the trial continues, whereas the standard delay does not. Thus, there comes a point in the trial at which the rational thing to do is to switch responding from the standard side, which has the shorter expected delay at trial onset, to the comparison (time-left) side, which has the shorter expected delay after some interval has elapsed. The only way to estimate this point—the elapsed interval (E) at which it pays to change over—is to subtract the elapsed interval from the fixed and known value of the comparison interval at the start of the trial.

Staddon and Higa's discussion of this experiment focuses on the question of the subjective value of the first half of a comparison interval versus the second half, which is not the proper focus. The most powerful result from these time-left experiments (as was stressed in the original publication by Gibbon & Church, 1981) is the relation between the midpoint of the cumulative changeover function (hereafter, the changeover point) and the absolute values of the comparison and standard intervals, when the ratio of these two reference intervals (the C/S ratio) is held constant. What matters is not *where* the changeover point is located for any particular values of C and S, which is what Staddon and Higa focus on. What matters is what happens to this changeover point as one increases the values of C and S proportionately (leaving their ratio unchanged). If subjective intervals are proportional to the logarithms of objective intervals, then the midpoint of the changeover distribution should be determined by the C/S ratio, which means that it should be independent of the actual values of C and S. This seems a priori exceedingly unlikely, and it is, in fact, contrary to experimental fact. Empirically, the changeover point increases linearly with the values of C and S. This result is fatal to the assumption that the magnitudes being subtracted in the head are the logarithms of the corresponding objective intervals.

In the end, Staddon and Higa seem to recognize the impossibility of explaining the time-left result while maintaining the assumption that the computation of the time left is carried out with quantities that are proportional to the logarithms of the intervals they represent. They write, "The claim is that no matter what the animal's internal code for elapsed time, it will also have some kind of compensatory perceptual constancy mechanism . . . that allows it to behave appropriately with respect to the real world (i.e., real time)" (p. 222). In Footnote 2, they write,

> Time may well be (and is, we contend) encoded nonlinearly, in the sense that it is mapped on to some internal variable that increases [sic] with elapsed time in a negatively accelerated way. Nevertheless, subjective time, like subjective weight and the other examples, is roughly proportional to real time. We argue that encoding determines experimental results that depend on discriminability, but subjective value determines results that depend on value (e.g., choice experiments).

They seem here to be making a distinction between how time is ''really'' encoded in the nervous system and how it is encoded when the animal has to do something that depends on its time estimates. In the latter case, they concede that ''subjective time . . . is roughly proportional to real time.'' It is in the nature of behavioral data that they can only be used to determine how a thing is represented at the point in the brain at which the signal that does the representing enters into a combinatorial computation that has behaviorally observable consequences.[2] The time-left experiment determines the relation between the subjective interval and the objective interval at the point in the behavior-generating process at which the brain determines the time left. At that point, the relation appears to be one of proportionality. Elsewhere in the brain where intervals are represented, the relation might have a different form. If so (and evidence of this remains to be found), then it will be difficult to defend the claim that the form at one point in the brain's processing is the ''real'' form, whereas the form elsewhere is the ''?'' form (virtual? imaginary? complex?—it is not clear what alternative to ''real'' would be appropriate to plug in here). The process that determines when the subject changes over from the standard option to the comparison option is presumably a real process, so the variables that enter into it are presumably themselves just as real (tangible, measurable, etc.).

Explaining the Distributions of Response Latencies

Thus, the question becomes whether there is evidence that at some point in some of the processes underlying at least some timing tasks, the form of the relation between the interval signal in the head and the objective interval is approximately logarithmic or, alternatively, a decay function. As indicated above, these are mutually exclusive hypotheses, because decay functions go to zero whereas the negative logarithm goes to minus infinity. The majority of the tasks to which SET has been applied are tasks that look at the distribution of response latencies relative to the reinforcement latency. SET does a good job of accounting for these distributions. By contrast, the observed distributions are not predicted by a model that assumes that (a) the relation between subjective intervals and objective intervals is either approximately logarithmic or is determined by a decay process and (b) the noise in a subjective interval is Gaussian with constant standard deviation. If the underlying measure of the ever-lengthening objective interval is really decaying to an asymptotic value of zero, as any decay model ought to assume, then it is going to be even more interesting to see what kind of assumptions about underlying noise will be required to explain the scalar variability that is so salient a property of the observed distributions. The particular MTS function that Staddon and Higa use in their Figure 1 is effectively zero by the time that 1,000 s have elapsed. Beyond that interval, there is nothing left to decay, so all objective intervals longer than that are subjectively the same. However, the likelihood of a pigeon's having resumed responding when a given proportion of the fixed interval in a fixed-interval schedule has elapsed is the same when the fixed interval between rewards is 30 s as when it is 3,000 s (Dews, 1970). This is a particularly striking and simple example of scalar variability in the timing of an operant response. How this could be explained by a sum of exponentials (or any other true decay function) and what the noise assumptions

[2] A computation in which the values of two different variables combine to determine the result. For example, in SET, the subjective measure of the currently elapsing interval is divided by the expectation retrieved from memory (a combinatorial operation) to produce the measure of similarity, which is compared to a threshold to determine whether the animal will or will not begin to respond. The comparison is also a combinatorial operation, namely, ordination (is the measure of similarity greater than the threshold). All of the basic operations of arithmetic (addition, subtraction, multiplication, division, and ordination) are combinatorial operations. Inversion, taking the log, raising to a power, and exponentiation are examples of noncombinatorial operations.

would have to be to get scalar variability over that range of intervals are very unclear.

Staddon and Higa do not make any suggestions about the sources or the form of the noise in the processes that generate timed responses. As a consequence, they do not offer an account of the distributions that are observed experimentally. Rather, they seem to argue that it is not reasonable to try to explain these distributions. If so, then there is little reason to offer alternatives to SET, because that is what SET principally explains. In refusing to try to explain the distributions of timed responses, Staddon and Higa abandon the field on which SET most often operates. More important, they avoid wrestling with some of the more difficult consequences of the assumption that time is measured in the head by a decay process. One of the more intractable problems with this assumption is reconciling it with the scalar variability observed in the distribution of response times.

At several points, Staddon and Higa seem to argue that SET is founded on the explanation of experiments on temporal discrimination. This assumption seems to underlie the paragraph (p. 223) whose second sentence begins, ''The fundamental flaw in the time-left argument is in fact *conceptual*'' and culminating in the sentence, ''The general point is that discriminability does not determine perceived value.'' This is a puzzling paragraph, because SET does not rest on the Fechnerian error of assuming that just noticeable differences are subjectively equal. In fact, it is well known that this assumption led to Staddon and Higa's sometime postulate (that the relation between the subjective quantity and the objective quantity is logarithmic), not to Gibbon's postulate. Indeed, the postulates that constitute SET are incompatible with the assumption that just noticeable differences are equal. Whatever SET's faults, the assumption that discriminability determines perceived value is not among them.

On the other hand, the failure to explain the scalar variability in the distributions of conditioned responses is a serious fault in Staddon and Higa's model. Scalar variability is a very well-established fact. It appears to be a manifestation of a deeper and broader principle, the principle of time-scale invariance. And, it appears to be irreconcilable with the assumption that temporal intervals are measured by a decay process, which is the central assumption in Staddon and Higa's model.

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